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DOE FUNDAMENTALS

CLASSICAL PHYSICS

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ABSTRACT

The *Classical Physics Fundamentals Handbook* was developed to assist nuclear facility operating contractors provide operators, maintenance personnel, and the technical staff with the necessary fundamentals training to ensure a basic understanding of physical forces and their properties. The handbook includes information on the units used to measure physical properties; vectors, and how they are used to show the net effect of various forces; Newton's Laws of motion, and how to use these laws in force and motion applications; and the concepts of energy, work, and power, and how to measure and calculate the energy involved in various applications. This information will provide personnel with a foundation for understanding the basic operation of various types of DOE nuclear facility systems and equipment.

Key Words: Training Material, Classical Physics, Vectors, Newton's Laws, Energy, Work, Power

CLASSICAL PHYSICS OVERVIEW

Department of Energy Fundamentals: *Classical Physics* was prepared as an information resource for personnel who are responsible for the operation of the Department's nuclear facilities. A basic understanding of classical physics is necessary for DOE nuclear facility operators, maintenance personnel, and the technical staff to safely operate and maintain the facility and facility support systems. The information in the handbook is presented to provide a foundation for applying engineering concepts to the job. This knowledge will help personnel more fully understand the impact that their actions may have on the safe and reliable operation of facility components and systems.

Classical Physics consists of five modules that are contained in one volume. The following is a brief description of the information presented in each module of the handbook.

Module 1 - Unit Systems

This module presents the concept of unit systems for the fundamental dimensions of mass, length, and time used in physics. Additionally, derivation of units and the conversion of these units is presented.

Module 2 - Vectors

This module contains information to aid in the determination of the net effect of various forces on an object and includes graphing vectors, adding vectors, and determining component vectors of a resultant vector.

Module 3 - Force and Motion

This module presents Newton's Laws of force and motion.

Module 4 - Application of Newton's Laws

This module describes the effect of static and dynamic forces on objects and includes a discussion of the forces commonly encountered in a nuclear facility.

Module 5 - Energy, Work, and Power

This module defines energy, work, and power, identifies their various forms, and discusses the conservation of energy, work, and power including the measurement and calculation of each.

The information contained in this document is by no means all encompassing. An attempt to present the entire science of classical physics would be impractical. However, *Classical Physics* does present enough information to provide the reader with a fundamental knowledge level sufficient to understand the advanced theoretical concepts presented in other subject areas, and to better understand basic system and equipment operations.

Department of Energy
Fundamentals Handbook

CLASSICAL PHYSICS
Module 1
Unit Systems

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- Murray, Raymond and Cobb, Grover, Physics Concepts and Consequences, Prentice-Hall, Inc., 1970.
- Alexander, Joseph, et al., Physics for Engineering Technology, John Wiley and Sons, 1978.
- Sears, Francis and Zemansky, Mark, University Physics, 3rd edition, Addison-Wesley Publishing Co.
- Science and Fundamental Engineering (SAFE), Classical Physics, "Measure" Windsor, CT: Combustion Engineering, Inc., 1988.
- Academic Program for Nuclear Power Plant Personnel, Volume II, General Physics Corporation, Library of Congress Card #A 397747, April 1982.

TERMINAL OBJECTIVE

1.0 Given appropriate conversion tables, **CONVERT** between English and SI system units of measurement.

ENABLING OBJECTIVES

1.1 **DEFINE** the three fundamental dimensions: length, mass, and time.

1.2 **LIST** standard units of the fundamental dimensions for each of the following systems:
- a. International System of Units (SI)
- b. English System

1.3 **DIFFERENTIATE** between fundamental and derived measurements.

1.4 Given appropriate conversion tables, **CONVERT** between English and SI units of length.

1.5 Given appropriate conversion tables, **CONVERT** between English and SI units of mass.

1.6 **CONVERT** time measurements between the following:
- a. Years
- b. Weeks
- c. Days
- d. Hours
- e. Minutes
- f. Seconds

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FUNDAMENTAL DIMENSIONS

Length, mass, and time are the three fundamental dimensions on which the measurement of all physical quantities is dependent.

EO 1.1 **DEFINE the three fundamental dimensions: length, mass, and time.**

EO 1.2 **LIST standard units of the fundamental dimensions for each of the following systems:**
a. International System of Units (SI)
b. English System

EO 1.3 **DIFFERENTIATE between fundamental and derived measurements.**

Fundamental Dimensions

Physics is a science based upon exact measurement of physical quantities that are dependent upon three fundamental dimensions. The three fundamental or primary dimensions are *mass, length,* and *time*. These three fundamental units must be understood in order to lay the foundation for the many concepts and principles presented in this material.

Mass

Mass is the amount of material present in an object. This measurement describes "how much" material makes up an object. Often, mass and weight are confused as being the same because the units used to describe them are similar. Weight (a derived unit, not a fundamental unit) is a measurement that describes the force of gravity on the "mass" of an object.

Length

Length is the distance between two points. The concept of length is needed to locate the position of a point in space and thereby describe the size of a physical object or system. When measuring a length of pipe, the ends of the pipe are the two points and the distance between the two points is the length. A typical unit used to describe length is the "foot."

Time

Time is the duration between two instants. The measurement of time is described in units of seconds, minutes, or hours.

Units

A number alone is not sufficient to describe a physical quantity. For example, to say that "a pipe must be 4 long to fit" has no meaning unless a unit of measurement for length is also specified. By adding units to the number, it becomes clear, "a pipe must be 4 feet long to fit."

The unit defines the magnitude of a measurement. If we have a measurement of length, the unit used to describe the length could be a foot or yard, each of which describes a different magnitude of length. The importance of specifying the units of a measurement for a number used to describe a physical quantity is doubly emphasized when it is noted that the same physical quantity may be measured using a variety of different units. For example, length may be measured in meters, inches, miles, furlongs, fathoms, kilometers, or a variety of other units.

Units of measurement have been established for use with each of the fundamental dimensions mentioned previously. The following section describes the unit systems in use today and provides examples of units that are used in each system.

Unit Systems

There are two unit systems in use at the present time, English units and International System of Units (SI).

In the United States, the English system is currently used. This system consists of various units for each of the fundamental dimensions or measurements. These units are shown in Table 1.

TABLE 1
English Units of Measurement

Length	Mass	Time
Inch	Ounce	* Second
* Foot	* Pound	Minute
Yard	Ton	Hour
Mile		Day
		Month
		Year

* Standard unit of measure

The English system is presently used in the field of engineering and throughout the United States. The foot-pound-second (FPS) system is the usual unit system used in the U.S. when dealing with physics.

Over the years there have been movements to standardize units so that all countries, including the United States, will adopt the SI system. The SI system is made up of two related systems, the meter-kilogram-second (MKS) system and the centimeter-gram-second (CGS) system.

The MKS and CGS systems are much simpler to use than the English system because they use a decimal-based system in which prefixes are used to denote powers of ten. For example, one kilometer is 1000 meters, and one centimeter is one one-hundredth of a meter. The English system has odd units of conversion. For example, a mile is 5280 feet, and an inch is one twelfth of a foot.

The MKS system is used primarily for calculations in the field of physics while both the MKS and CGS systems are used in the field of chemistry. The units for each of these systems are shown in Tables 2 and 3 below.

TABLE 2
MKS Units of Measurement

Length	Mass	Time
Millimeter	Milligram	* Second
* Meter	Gram	Minute
Kilometer	* Kilogram	Hour
		Day
		Month
		Year

* Standard unit of measure

TABLE 3
CGS Units of Measurement

Length	Mass	Time
* Centimeter	Milligram	* Second
Meter	* Gram	Minute
Kilometer	Kilogram	Hour
		Day
		Month
		Year

* Standard unit of measure

The following tables show approximate lengths, masses, and times for some familiar objects or events.

TABLE 4
Approximate Lengths of Familiar Objects

Object	Length (meters)
Diameter of Earth Orbit Around Sun	2×10^{11}
Football Field	1×10^{2}
Diameter of Dime	2×10^{-2}
Thickness of Window Pane	1×10^{-3}
Thickness of Paper	1×10^{-4}

TABLE 5
Approximate Masses of Familiar Objects

Object	Mass (kilograms)
Earth	6×10^{24}
House	2×10^{5}
Car	2×10^{3}
Quart of Water	1
Dime	3×10^{-3}
Postage Stamp	5×10^{-8}

TABLE 6
Approximate Times of Familiar Events

Event	Time (seconds)
Age of Earth	2×10^{17}
Human Life Span	2×10^{9}
Earth Rotation Around Sun	3×10^{7}
Earth Rotation Around Axis	8.64×10^{4}
Time Between Heart Beats	1

Derived Measurements

Most physical quantities have units that are combinations of the three *fundamental* dimensions of length, mass, and time. When these dimensions or measurements are combined, they produce what are referred to as *derived* units. This means that they have been "derived" from one or more fundamental measurements. These combinations of fundamental measurements can be the combination of the same or different units. The following are examples of various derived units.

Area

Area is the product of two lengths (e.g., width x length for a rectangle); thus, it has the units of length squared, such as square inches (in.^{2}) or square meters (m^2).

$1 \text{ m} \times 1 \text{ m} = 1 \text{ m}^2$

$4 \text{ in.} \times 2 \text{ in.} = 8 \text{ in.}^2$

Volume

Volume is the product of three lengths (e.g., length x width x depth for a rectangular solid); thus, it has the units of length cubed, such as cubic inches (in.^{3}) or cubic meters (m^3). The MKS and CGS unit systems have a specific unit for volume called the liter (l). One liter is equal to 1000 cubic centimeters (1 l = 1000 cm^3).

$2 \text{ in.} \times 3 \text{ in.} \times 5 \text{ in.} = 30 \text{ in.}^3$

Density

Density is a measure of the mass of an object per unit volume; thus, it has units of mass divided by length cubed such as kilograms per cubic meter (kg/m^3) or pounds per cubic foot (lbs/ft^3).

$$15\ lbs/5\ ft^3 = 3\ lbs/ft^3$$

Velocity

Velocity is the change in length per unit time; thus, it has units such as kilometers per hour (km/h) or feet per second (ft/s).

Acceleration

Acceleration is a measure of the change in velocity or velocity per unit time; thus, it has units such as centimeters per second per second (cm/s^2) or feet per second per second (ft/s^2).

Summary

The main points of this chapter are summarized below.

Fundamental Dimensions Summary

The fundamental measurements consist of:

- Length - distance between two points
- Mass - amount of material in an object
- Time - duration between two instants

The English system of units is based on the following standard units:

- Foot
- Pound
- Second

The SI system of measurement consists of the following standard units:

MKS	CGS
• Meter	• Centimeter
• Kilogram	• Gram
• Second	• Second

Derived units are made up of a combination of units to describe various physical quantities. For example:

- Area - square inches (in.^{2})
- Volume - cubic inches (in.^{3}) or liters
- Density - mass per volume (lb/in.^{3})

UNIT CONVERSIONS

In order to apply measurements from the SI system to the English system, it is necessary to develop relationships of known equivalents (conversion factors). These equivalents can then be used to convert from the given units of measure to the desired units of measure.

EO 1.4 CONVERT between English and SI units of length.

EO 1.5 CONVERT between English and SI units of mass.

EO 1.6 CONVERT time measurements between the following:

a. Years
b. Weeks
c. Days
d. Hours
e. Minutes
f. Seconds

Personnel at DOE nuclear facilties are often exposed to both the English and SI systems of units in their work. In some cases, the measurements that are taken or read from an instrument will be different from those required by a procedure. This situation will require the conversion of measurements to those required by the procedure.

Conversion Factors

Conversion factors are based on relationships of equivalents from different measurement systems. These conversion factors are then applied to the given measurement in order to convert it to the units that are required. The equivalent relationships between different units of measurement are defined in conversion tables. Some examples from conversion tables are given below.

- 1 yard = .9144 meters
- 1 kilogram = 2.205 pounds mass (lbm)
- 1 hour = 3600 seconds

A typical conversion table is shown in Table 7.

TABLE 7
Conversion Table

Length	1 yd	=	0.9144 m
	12 in.	=	1 ft
	5280 ft	=	1 mile
	1 m	=	3.281 ft
	1 in.	=	0.0254 m
Time	60 sec	=	1 min
	3600 sec	=	1 hr
Mass	1 lbm	=	0.4535 kg
	2.205 lbm	=	1 kg
	1 kg	=	1000 g
Area	1 ft^2	=	144 $in.^2$
	10.764 ft^2	=	1 m^2
	1 yd^2	=	9 ft^2
	1 $mile^2$ =		3.098 X 10^6 yd^2
Volume	7.48 gal	=	1 ft^3
	1 gal	=	3.785 l (liter)
	1 l	=	1000 cm^3

Unit Conversion

To convert from one measurement unit to another measurement unit (for example, to convert 5 feet to inches), first select the appropriate equivalent relationship from the conversion table (for this example, 1 foot = 12 inches). Conversion is basically a multiplication by 1. We can divide both sides of the equation 1 ft = 12 inches by 1 foot to obtain the following.

$$\text{Then} \quad \frac{1\ \text{ft}}{1\ \text{ft}} = \frac{12\ \text{inches}}{1\ \text{foot}} \quad \text{or} \quad 1 = \frac{12\ \text{inches}}{1\ \text{foot}}$$

The relationship $\frac{12\ \text{inches}}{1\ \text{foot}}$ is a conversion factor which we can use in our example to convert 5 feet to inches.

Using the example, 5 feet is to be converted to inches. Start with the obvious equation

5 ft = 5 ft, and multiply the right hand side by $1 = \frac{12 \text{ inches}}{\text{foot}}$.

5 feet = 5 feet x 1

$$5 \text{ feet} = 5 \text{ feet} \times \frac{12 \text{ inches}}{\text{ft}} = 5 \times 12 \text{ inches} = 60 \text{ inches.}$$

Thus, 5 feet is equivalent to 60 inches.

Steps for Unit Conversion

Using the following example, we will step through the process for converting from a given set of units to a desired set of units.

Convert 795 m to ft.

Step 1. Select the equivalent relationship from the conversion table (Table 7).

1 meter = 3.281 ft

Step 2. Divide to obtain the factor 1 as a ratio $\left(\frac{\text{desired units}}{\text{present units}}\right)$.

$$1 = \frac{3.281 \text{ ft}}{1 \text{ m}}$$

Step 3. Multiply the quantity by the ratio.

$$(795 \text{ m}) \text{ x } \left(\frac{3.281 \text{ ft}}{1 \text{ m}}\right) = \left(\frac{795 \text{ m}}{1}\right) \text{ x } \left(\frac{3.281 \text{ ft}}{1 \text{ m}}\right)$$

= 795 x 3.281 ft

= 2608.395 ft

If an equivalent relationship between the given units and the desired units cannot be found in the conversion tables, multiple conversion factors must be used. The conversion is performed in several steps until the measurement is in the desired units. The given measurement must be multiplied by each conversion factor (ratio). After the common units have been canceled out, the answer will be in the desired units.

Example: Convert 2.91 sq miles to sq meters.

Step 1. Select the equivalent relationship from the conversion table. Because there is no direct conversion shown for square miles to square meters, multiple conversions will be necessary. For this example the following conversions will be used.

sq miles to sq yds to sq ft to sq m

1 sq mile = 3.098×10^6 sq yd

1 sq yd = 9 sq ft

10.764 sq ft = 1 sq m

Step 2. Express the relationship as a ratio (desired unit/present unit).

$$1 = \frac{3.098 \times 10^6 \text{ sq yd}}{1 \text{ sq mile}}$$

Step 3. Multiply the quantity by the ratio.

$$(2.91 \text{ sq miles}) \times \left(\frac{3.098 \times 10^6 \text{ sq yd}}{1 \text{ sq mile}}\right) = 9.015 \times 10^6 \text{ sq yd}$$

Step 4. Repeat the steps until the value is in the desired units.

$$1 = \frac{9 \text{ sq ft}}{1 \text{ sq yd}}$$

$$(9.015 \times 10^6 \text{ sq yd}) \times \left(\frac{9 \text{ sq ft}}{1 \text{ sq yd}}\right) = 8.114 \times 10^7 \text{ sq ft}$$

$$1 = \frac{1 \text{ sq m}}{10.764 \text{ sq ft}}$$

$$(8.114 \text{ x } 10^7 \text{ sq ft}) \text{ x } \left(\frac{1 \text{ sq m}}{10.764 \text{ sq ft}}\right) = \frac{(8.114 \text{ x } 10^7)\ (1 \text{ sq m})}{10.764}$$

$$= \frac{8.114 \text{ x } 10^7 \text{ sq m}}{10.764}$$

$$= 7.538 \text{ x } 10^6 \text{ sq m}$$

It is possible to perform all of the conversions in a single equation as long as all of the appropriate conversion factors are included.

$$(2.91 \text{ sq miles}) \text{ x } \left(\frac{3.098 \text{ x } 10^6 \text{ sq yd}}{1 \text{ sq mile}}\right) \text{ x } \left(\frac{9 \text{ sq ft}}{1 \text{ sq yd}}\right) \text{ x } \left(\frac{1 \text{ sq m}}{10.764 \text{ sq ft}}\right)$$

$$= \frac{(2.91) \text{ x } (3.098 \text{ x } 10^6)\ (9)\ (1 \text{ sq m})}{10.764}$$

$$= \frac{8.114 \text{ x } 10^7 \text{ sq m}}{10.764}$$

$$= 7.538 \text{ x } 10^6 \text{ sq m}$$

Example:

A Swedish firm is producing a valve that is to be used by an American supplier. The Swedish firm uses the MKS system for all machining. To conform with the MKS system, how will the following measurements be listed?

Valve stem	57.20 in.
Valve inlet and outlet	
I.D.	22.00 in.
O.D.	27.50 in.

Solution:

Valve stem
57.20 in. x 0.0254 m/in. = 1.453 m

Valve inlet and outlet
I.D. 22.00 x 0.0254 = 0.559 m
O.D. 27.50 x 0.0254 = 0.699 m

Examples of common conversion factors are shown in Table 8.

TABLE 8
Conversion Factors

CONVERSION FACTORS FOR COMMON UNITS OF MASS

		g	kg	t	lbm
1 gram	=	1	0.001	10^{-6}	2.2046×10^{-3}
1 kilogram	=	1000	1	0.001	2.2046
1 metric ton (t)	=	10^{6}	1000	1	2204.6
1 pound-mass (lbm)	=	453.59	0.45359	4.5359×10^{-4}	1
1 slug	=	14,594	14.594	0.014594	32.174

CONVERSION FACTORS FOR COMMON UNITS OF LENGTH

		cm	m	km	in.	ft	mi
1 centimeter	=	1	0.01	10^{-5}	0.39370	0.032808	$6.2137 \text{ X } 10^{-6}$
1 meter	=	100	1	0.001	39.370	3.2808	6.2137×10^{-4}
1 kilometer	=	10^{5}	1000	1	39,370	3280.8	0.62137
1 inch	=	2.5400	0.025400	2.5400×10^{-5}	1	0.083333	1.5783×10^{-5}
1 foot	=	30.480	0.30480	3.0480×10^{-4}	12.000	1	1.8939×10^{-4}
1 mile	=	1.6093×10^{5}	1609.3	1.6093	63,360	5280.0	1

TABLE 8 (Cont.)
Conversion Factors

CONVERSION FACTORS FOR COMMON UNITS OF TIME

		sec	min	hr
1 second	=	1	0.017	2.7×10^{-4}
1 minute	=	60	1	0.017
1 hour	=	3600	60	1
1 day	=	86,400	1440	24
1 year	=	3.15×10^{7}	5.26×10^{5}	8760

		day	year
1 second	=	1.16×10^{-5}	3.1×10^{-8}
1 minute	=	6.9×10^{-4}	1.9×10^{-6}
1 hour	=	4.16×10^{-2}	1.14×10^{-4}
1 day	=	1	2.74×10^{-3}
1 year	=	365	1

Summary

Unit conversion is summarized below.

Unit Conversion Summary

- Conversion Tables list equivalent relationships.
- Conversion Factors are obtained by dividing to get a multiplying factor (1).

Unit Conversion Steps

- Step 1 - Select the equivalent relationship from the conversion table.
- Step 2 - Express the relationship as a conversion factor.
- Step 3 - Multiply the given quantity by the conversion factor.

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CLASSICAL PHYSICS
Module 2
Vectors

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- Murray, Raymond and Cobb, Grover, Physics Concepts and Consequences, Prentice-Hall, Inc., 1970.
- Alexander, Joseph, et al., Physics for Engineering Technology, John Wiley and Sons, 1978.
- Sears, Francis and Zemansky, Mark, University Physics, 3rd edition, Addison-Wesley Publishing Co.
- Science and Fundamental Engineering (SAFE), Classical Physics, "Measure" Windsor, CT: Combustion Engineering, Inc., 1988.
- Academic Program for Nuclear Power Plant Personnel, Volume II, General Physics Corporation, Library of Congress Card #A 397747, April 1982.

TERMINAL OBJECTIVE

1.0 Using vectors, **DETERMINE** the net force acting on an object.

ENABLING OBJECTIVES

1.1 **DEFINE** the following as they relate to vectors:
- a. Scalar quantity
- b. Vector quantity
- c. Vector component
- d. Resultant

1.2 **DETERMINE** components of a vector from a resultant vector.

1.3 **ADD** vectors using the following methods:
- a. Graphical
- b. Component addition
- c. Analytical

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SCALAR AND VECTOR QUANTITIES

Scalars are quantities that have magnitude only; they are independent of direction. Vectors have both magnitude and direction. The length of a vector represents magnitude. The arrow shows direction.

EO 1.1 DEFINE the following as they relate to vectors:
a. Scalar quantity
b. Vector quantity

Scalar Quantities

Most of the physical quantities encountered in physics are either scalar or vector quantities. A *scalar* quantity is defined as a quantity that has magnitude only. Typical examples of scalar quantities are time, speed, temperature, and volume. A scalar quantity or parameter has no directional component, only magnitude. For example, the units for time (minutes, days, hours, etc.) represent an amount of time only and tell nothing of direction. Additional examples of scalar quantities are density, mass, and energy.

Vector Quantities

A *vector* quantity is defined as a quantity that has both magnitude and direction. To work with vector quantities, one must know the method for representing these quantities.

Magnitude, or "size" of a vector, is also referred to as the vector's "displacement." It can be thought of as the scalar portion of the vector and is represented by the length of the vector. By definition, a vector has both magnitude and direction. Direction indicates how the vector is oriented relative to some reference axis, as shown in Figure 1.

Using north/south and east/west reference axes, vector "A" is oriented in the NE quadrant with a direction of 45º north of the EW axis. G iving direction to scalar "A" makes it a vector. The length of "A" is representative of its magnitude or displacement.

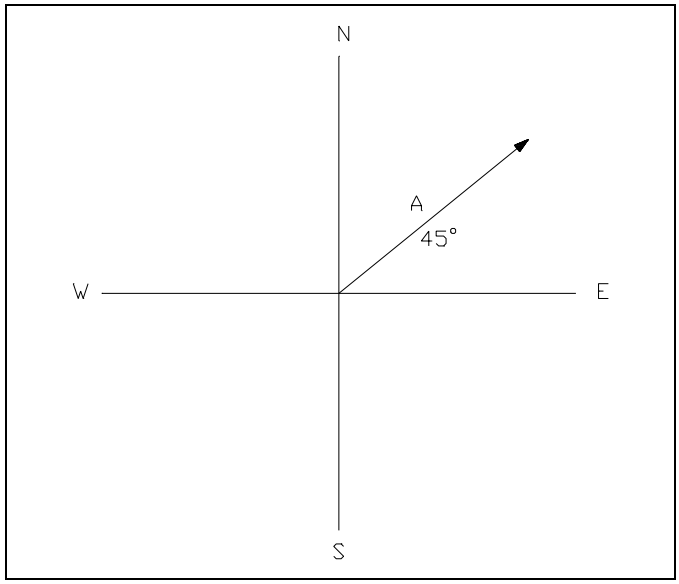

Figure 1 Vector Reference Axis

To help distinguish between a scalar and a vector, let's look at an example where the only information known is that a car is moving at 50 miles per hour. The information given (50 mph) only refers to the car's speed, which is a scalar quantity. It does not indicate the direction the car is moving. However, the same car traveling at 50 mph due east indicates the velocity of the car because it has magnitude (50 mph) and direction (due east); therefore, a vector is indicated. When a vector is diagrammed, a straight line is drawn to show the unit of length. An arrow is drawn on one end of the line. The length of the line represents the magnitude of the vector, and the arrow represents the direction of the vector.

Description of a Simple Vector

Vectors are simple straight lines used to illustrate the direction and magnitude of certain quantities. Vectors have a starting point at one end (tail) and an arrow at the opposite end (head), as shown in Figure 2.

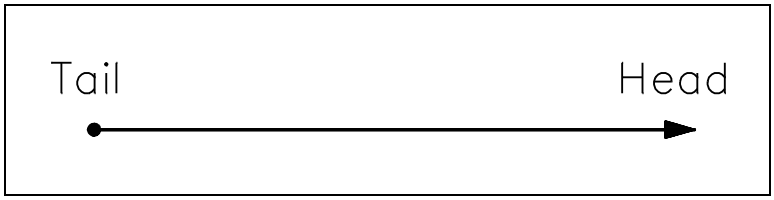

Figure 2 Vector

Examples of Vector Quantities

Displacement, velocity, acceleration, and force are examples of vector quantities. Momentum and magnetic field strength are also good examples of vector quantities, although somewhat more difficult to understand. In each of these examples, the main ingredients of magnitude and direction are present.

Summary

The important aspects of scalar and vector quantities are summarized below.

Scalar and Vector Quantities Summary

SCALAR QUANTITIES	VECTOR QUANTITIES
• Magnitude only	• Both magnitude and direction
• Independent of direction	• Length represents magnitude
• Examples of scalars include: time, speed, volume, and temperature	• Arrow shows direction
	• Examples of vectors include: force, velocity, and acceleration

VECTOR IDENTIFICATION

Vectors are symbolized in specific ways in texts and on graphs, using letters or rectangular coordinates.

In Written Materials

In textbooks, vector quantities are often represented by simply using a boldfaced letter (**A, B, C, R**). Particular quantities are predefined (**F** - force, **V** - velocity, and **A** - acceleration). Vector quantities are sometimes represented by $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, $\vec{\mathbf{C}}$, $\vec{\mathbf{R}}$. Regardless of the convention used, specific vector quantities must include magnitude and direction (for example, 50 mph due north, or 50 lbf at 90º).

Graphic Representation

Vector quantities are graphically represented using the rectangular coordinate system, a two-dimensional system that uses an x-axis and a y-axis. The x-axis is a horizonal straight line. The y-axis is a vertical straight line, perpendicular to the x-axis. An example of a rectangular system is shown in Figure 3.

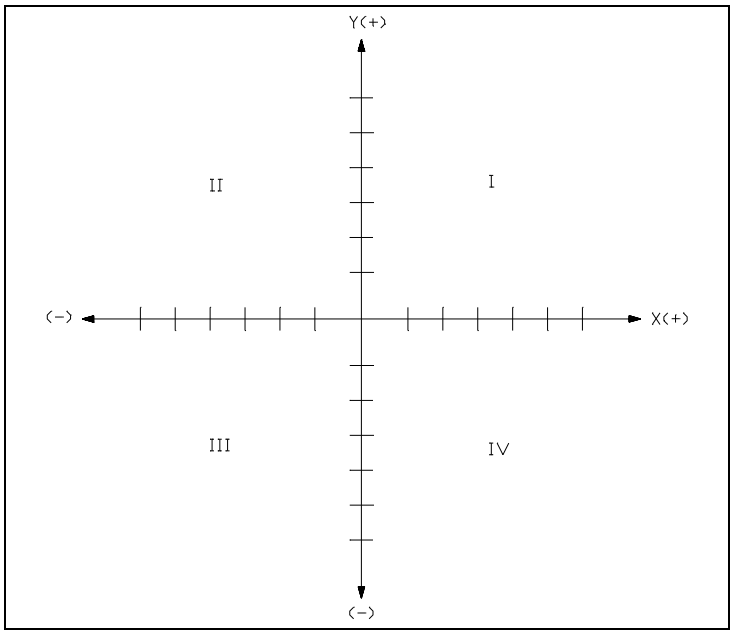

Figure 3 Rectangular Coordinate System

The intersection of the axes is called the point of origin. Each axis is marked off in equal divisions in all four directions from the point of origin. On the horizonal axis (x), values to the right of the origin are positive (+). Values to the left of the origin are negative (-). On the vertical axis (y), values above the point of origin are positive (+). Values below the origin are negative (-). It is very important to use the same units (divisions) on both axes.

The rectangular coordinate system creates four infinite quadrants. Quadrant I is located above and to the right of the origin. Quadrant II is located above and to the left of the origin. Quadrant III is situated to the left and below the origin, and quadrant IV is located below and to the right of the origin (see Figure 3).

Graphic Representation of Vectors

With the coordinate system defined, the following explanation will illustrate how to locate vectors in that system.

First, using a ruler and graph paper, a rectangular coordinate system is laid out as described in the previous section. The x- and y-axes are labeled. Equal divisions are marked off in all four directions. Those to the right and above the point of origin are labeled positive (+). Those to the left and below the point of origin are labeled negative (-).

Beginning at the point of origin (intersection of the axes), a line segment of the proper length is shown along the x-axis, in the positive direction. This line segment represents the vector magnitude, or displacement. An arrow is placed at the "head" of the vector to indicate direction. The "tail" of the vector is located at the point of origin (see Figure 4).

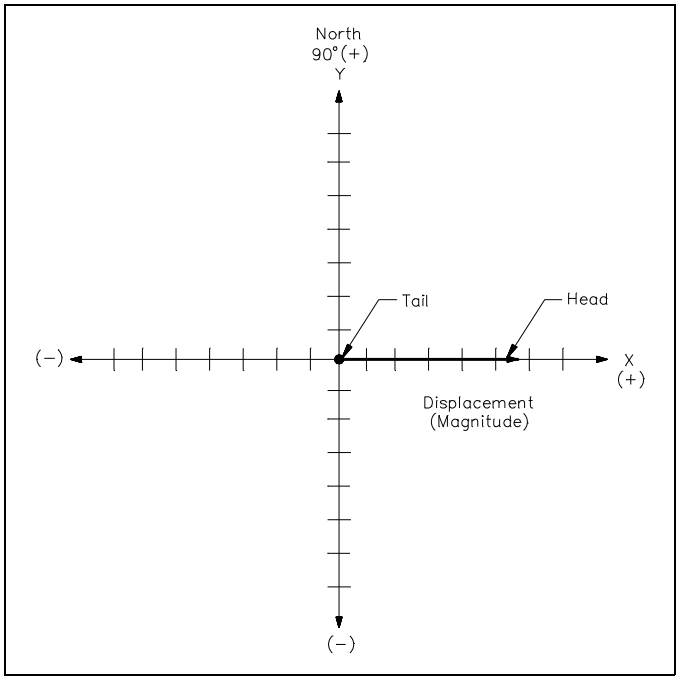

Figure 4 Displaying Vectors Graphically - Magnitude

When vectors are drawn that do not fall on the x- or y-axes, the tail is located at the point of origin. Depending on the vector description, there are two methods of locating the head of the vector. If coordinates (x,y) are given, these values can be plotted to locate the vector head. If the vector is described in degrees, the line segment can be rotated counterclockwise from the x-axis to the proper orientation, as shown in Figure 5.

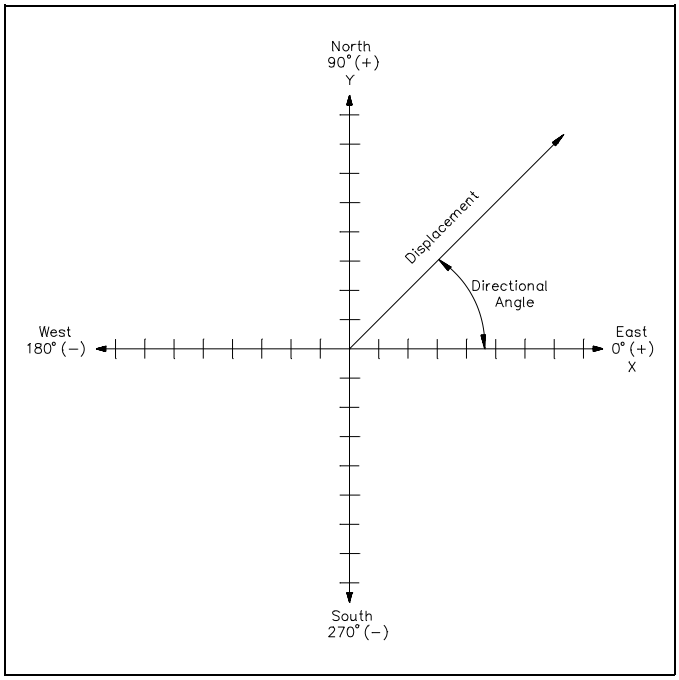

Figure 5 Display Vectors Graphically - Direction

Because the x- and y-axes define direction, conventional directional coordinates and degrees may also be used to identify the x- and y-axes (see Figures 6 and 7).

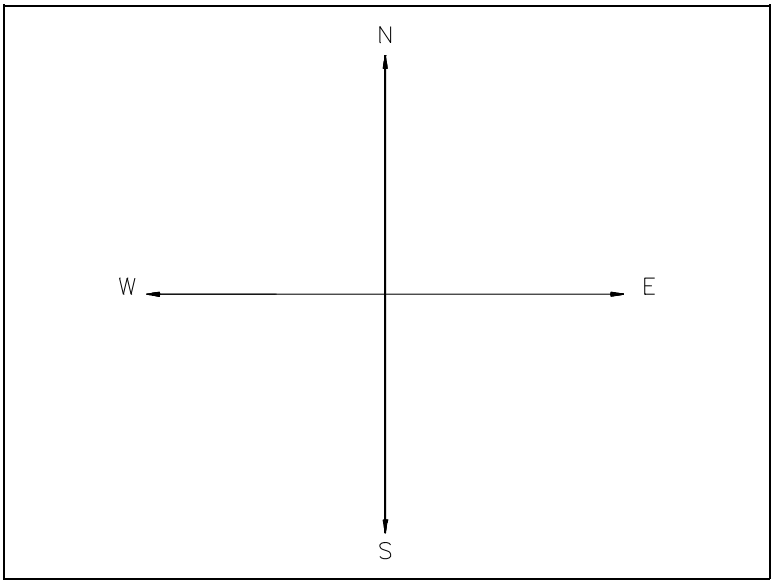

Figure 6 Directional Coordinates

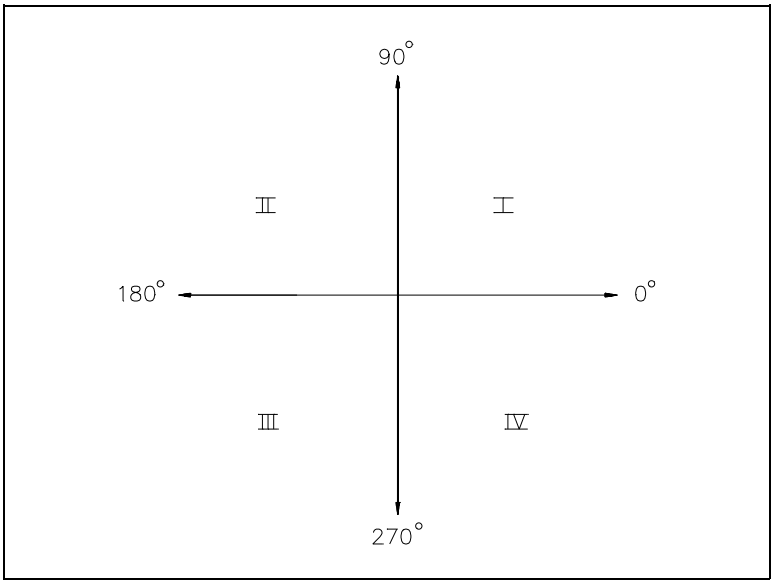

Figure 7 Degree Coordinates

Summary

The main points covered in this chapter are summarized below.

Vector Identification Summary

- In text:
  - Boldfaced letters (**A**, **F**, **R**)
  - Capital letters with arrows over ($\vec{\mathbf{A}}$, $\vec{\mathbf{F}}$, $\vec{\mathbf{R}}$)
- Graphically:
  - (x,y) coordinates
  - Directional Coordinates
  - Degrees

VECTORS: RESULTANTS AND COMPONENTS

A resultant is a single vector which represents the combined effect of two or more other vectors (called components). The components can be determined either graphically or by using trigonometry.

EO 1.1 DEFINE the following as they relate to vectors:
c. Vector component
d. Resultant

EO 1.2 DETERMINE components of a vector from a resultant vector.

Resultant

When two or more vectors are added they yield the sum or *resultant* vector. A resultant vector is the result or sum of vector addition. Vector addition is somewhat different from addition of pure numbers unless the addition takes place along a straight line. In the latter case, it reduces to the number line of standards or scale addition. For example, if one walks five miles east and then three miles east, he is eight miles from his starting point. On a graph (Figure 8), the sum of the two vectors, i.e., the sum of the five miles plus the three mile displacement, is the total or resultant displacement of eight miles.

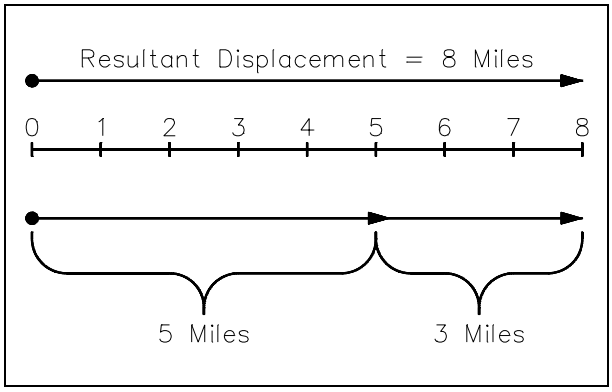

Figure 8 Vector Addition in Same Direction

Similarly, if one walks five miles east and then three miles west, the resultant displacement is two miles east (Figure 9).

The vector diagrams of Figure 8 and Figure 9 are basically scale diagrams of what is happening in the real world of addition of vector quantities.

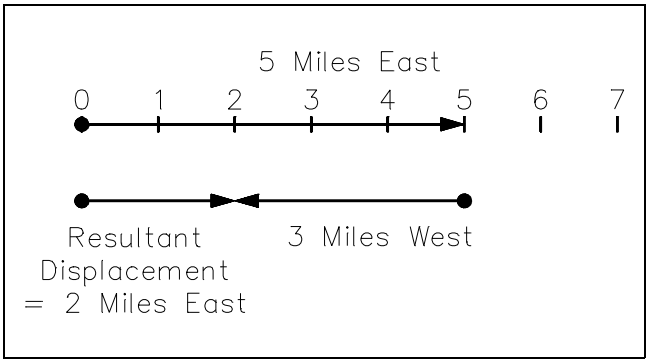

Figure 9 Vector Addition in Opposite Directions

Consider next the addition of vector quantities which are not in a straight line. For example, consider the resultant displacement when a person travels four miles east and then three miles north. Again a scale drawing (Figure 10) is in order. Use a scale of 1 inch = 1 mile.

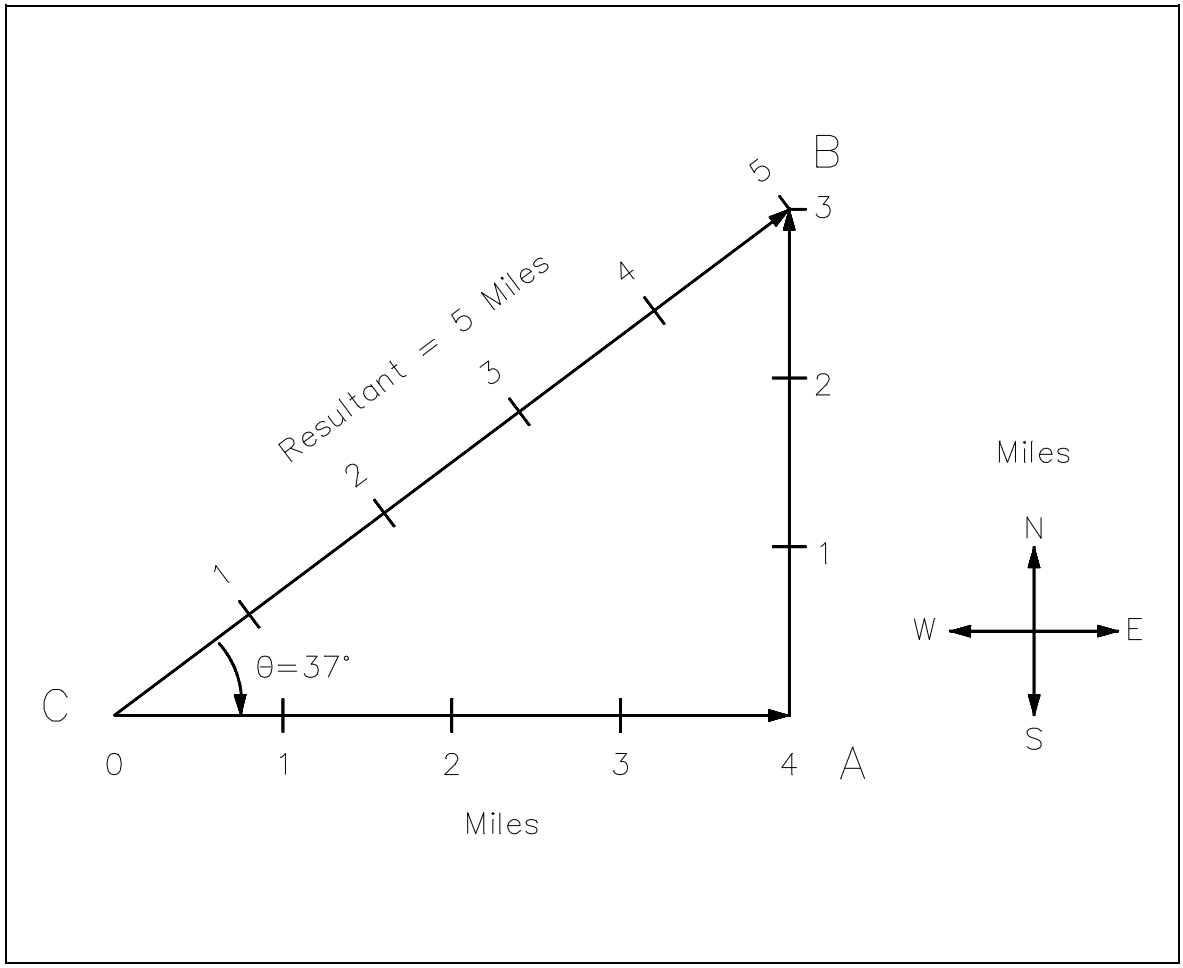

Figure 10 Vector Addition Not in Same Line

When drawing a scale drawing, one draws a straight line from the origin C to the final position B to represent the net or resultant displacement. Drawing the straight line CB and measuring its length, one should obtain about 5 inches. Then, since the scale of the drawing is 1 inch = 1 mile, this is used as a conversion factor giving 5 inches $\times \frac{1 \text{ mile}}{\text{inch}}$ = 5 miles as the displacement.

Using a protractor or trigonometry, the acute angle ACB can be determined to be about 37°. Thus, the resultant (or vector sum) of traveling 4 miles east plus 3 miles north is a displacement of 5 miles at 37 degrees north of east.

It is left as an exercise for the student to show that vector addition is commutative, using the above example. Specifically, make a scale drawing showing that traveling 3 miles north and then 4 miles east yields the same resultant as above.

It is also reasonably obvious that more than two vectors can be added. One can travel three miles east and then three miles north and then three miles west and arrive at a point three miles north of the starting point. The sum of these three displacements is a resultant displacement of three miles north. (If this is not immediately apparent, sketch it.)

A student problem is to find the net or resultant displacement if a person travels 9 miles south and then 12 miles east and then 25 miles north. Make a scale drawing and determine the magnitude and direction of the resultant displacement. A scale of 2 miles per centimeter or 4 miles per inch will fit the drawing on standard paper.

Answer: About 20 miles at 53° north of east.

Vector Components

Components of a vector are vectors, which when added, yield the vector. For example, as shown in the previous section (Figure 10), traveling 3 miles north and then 4 miles east yields a resultant displacement of 5 miles, 37° north of east. This example demonstrates that component vectors of any two non-parallel directions can be obtained for any resultant vector in the same plane. For the purposes of this manual, we restrict our discussions to two dimensional space. The student should realize that vectors can and do exist in three dimensional space.

One could write an alternate problem: "If I am 5 miles from where I started northeast along a line 37° N of east, how far north and how far east am I from my original position?" Drawing this on a scale drawing, the vector components in the east and north directions can be measured to be about 4 miles east and 3 miles north. These two vectors are the components of the resultant vector of 5 miles, 37° north of east.

Component vectors can be determined by plotting them on a rectangular coordinate system. For example, a resultant vector of 5 units at 53° can be broken down into its respective x and y magnitudes. The x value of 3 and the y value of 4 can be determined using trigonometry or graphically. Their magnitudes and position can be expressed by one of several conventions including: (3,4), (x=3, y=4), (3 at 0°, 4 at 90°), and (5 at 53°). In the first expression, the first term is the x-component (F_x), and the second term is the y-component (F_y) of the associated resultant vector.

As in the previous example, if only the resultant is given, instead of component coordinates, one can determine the vector components as illustrated in Figure 11. First, plot the resultant on rectangular coordinates and then project the vector coordinates to the axis. The length along the x-axis is F_x, and the length along the y-axis is F_y. This method is demonstrated in the following example.

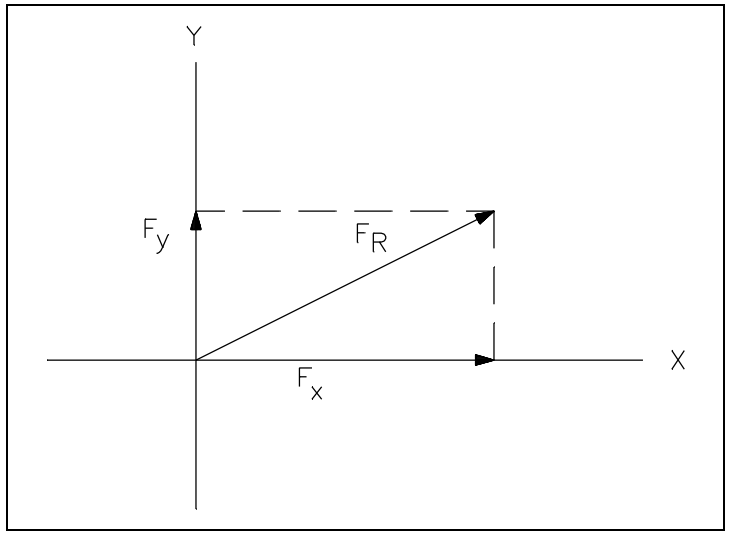

Figure 11 Vector Components

For the resultant vector shown in Figure 12, determine the component vectors given F_R = 50 lbf at 53°. First, project a perpendicular line from the head of F_R to the x-axis and a similar line to the y-axis. Where the projected lines meet, the axes determine the magnitude size of the component vectors. In this example, the component vectors are 30 lbf at 0° (F_x) and 40 lbf at 90° (F_y). If F_R had not already been drawn, the first step would have been to draw the vector.

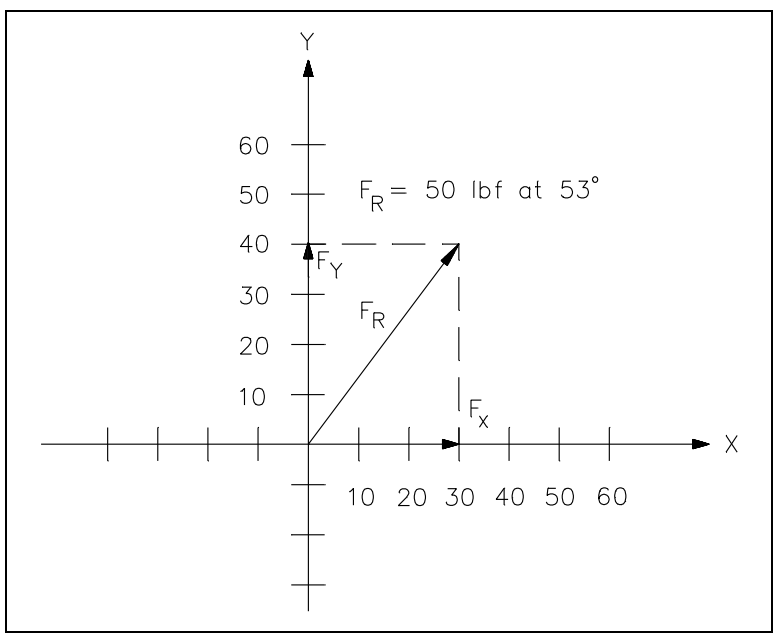

Figure 12 Component Vectors

As an exercise, the student should graphically find the easterly and northerly components of a 13 mile displacement at 22.6° north of east. The correct answer is 5 miles east and 12 miles north.

Trigonometry may also be used to determine vector components. Before explaining this method, it may be helpful to review the fundamental trigonometric functions. Recall that trigonometry is a branch of mathematics that deals with the relationships between angles and the length of the sides of triangles. The relationship between an acute angle of a right triangle, shown in Figure 13, and its sides is given by three ratios.

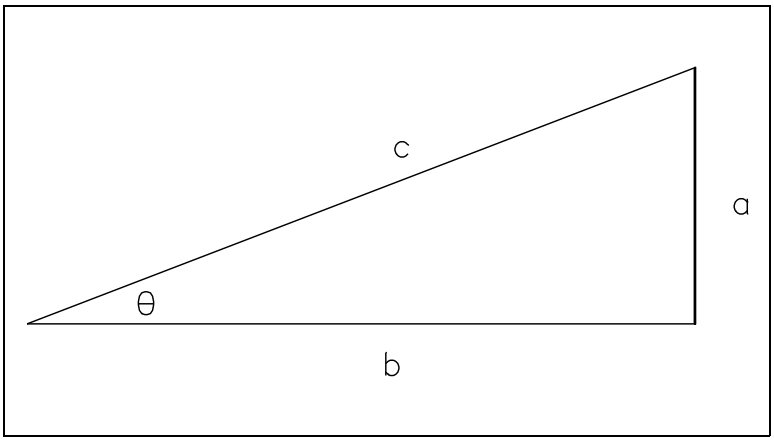

Figure 13 Right Triangle

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c} \tag{2-1}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c} \tag{2-2}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b} \tag{2-3}$$

Before attempting to calculate vector components, first make a rough sketch that shows the approximate location of the resultant vector in an x-y coordinate system. It is helpful to form a visual picture before selecting the correct trigonometric function to be used. Consider the example of Figure 12, that was used previously. This time the component vectors will be calculated.

Example 1: Determine the component vectors, F_x and F_y, for F_R = 50 lbf at 53º in Figure 14. Use trigonometric functions.

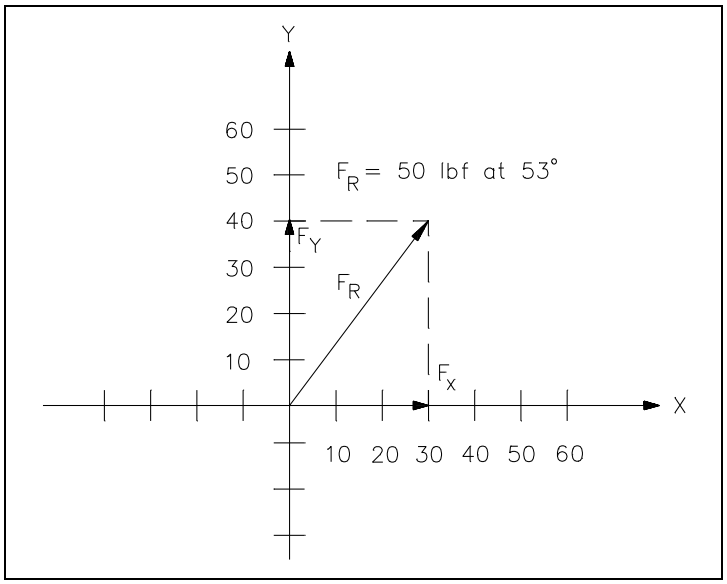

Figure 14 F_R = 50 lbf at 53º

F_x is calculated as follows:

$\cos\theta$ = adjacent/hypotenuse
$\cos\theta = F_x/F_R$ or $F_x = F_R \cos\theta$
$F_x = (50)(\cos 53^o)$
$F_x = (50)(0.6018)$
F_x = 30 lbf on x-axis

F_y is calculated as follows:

$\sin\theta$ = opposite/hypotenuse
$\sin\theta = F_y/F_R$ or $F_y = F_R \sin\theta$
$F_y = (F_R)(\sin\theta)$
$F_y = (50)(\sin 53^o)$
$F_y = (50)(0.7986)$
F_y = 40 lbf on y-axis

Therefore, the components for F_R are F_x = 30 lbf at 0º and F_y = 40 lbf at 90º. Note that this result is identical to the result obtained using the graphic method.

Example 2: What are the component vectors, given F_R = 80 lbf at 220º? See Figure 15.

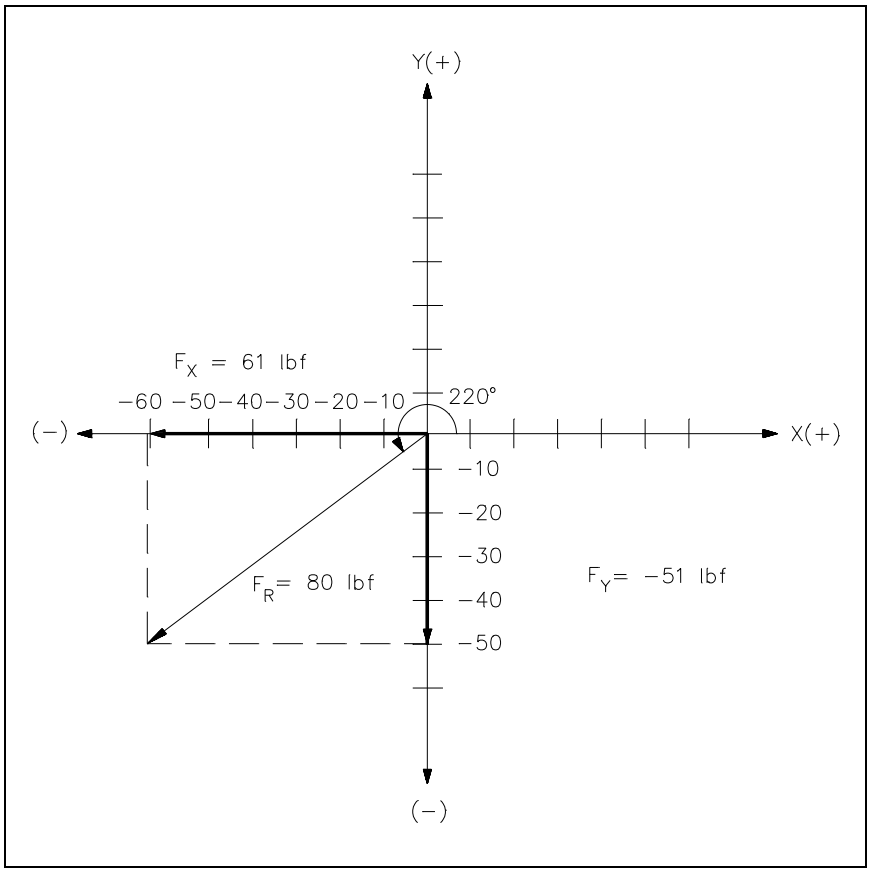

Figure 15 F_R = 80 lbf at 220º

F_x is calculated as follows:

$\cos\theta$ = adjacent/hypotenuse

$\cos\theta = F_x/F_R$ or $F_x = F_R \cos\theta$

$F_x = (F_R)(\cos\theta)$

$F_x = (80)(\cos 220^o)$

$F_x = (80)(-0.766)$

F_x = -61 lbf at 0º or 61 lbf at 180º

F_y is calculated as follows:

$\sin\theta$ = opposite/hypotenuse
$\sin\theta = F_y/F_R$ or $F_y = F_R \sin\theta$
$F_y = F_R \sin\theta$
$F_y = (80)(\sin 220^o)$
$F_y = (80)(-0.6428)$
$F_y = -51$ lbf at 90° or 51 lbf at 270°

Therefore, the components for F_R are $F_x = 61$ at 180° and $F_y = 51$ lbf at 270°.

Summary

Vector terminology is summarized below.

Vector Terminology Summary

- A resultant is a single vector that can replace two or more vectors.
- Components can be obtained for any two non-parallel directions if the vectors are in the same plane. Restricting the treatment to perpendicular directions and two dimensional space, the components of a vector are the two vectors in the x and y (or east-west and north-south) directions which produce the same effect as the original vector (or add to produce the original vector).
- Components are determined from data, graphically or analytically.

GRAPHIC METHOD OF VECTOR ADDITION

Vectors are added to determine the magnitude and direction of the resultant.

EO 1.3 ADD vectors using the following methods:
a. Graphical

Vector Addition

Component vectors are added to determine the resultant vector. For example, when two or more forces are acting on a single object, vector addition is used to determine the direction and magnitude of the net (resultant) force on the object. Consider an airplane that travels due east for 100 miles at 500 mph, then NE for 50 miles at 400 mph, and finally north for 500 miles at 500 mph. Vector addition can be used to determine the net distance the airplane is from its point of origin or to predict when it will arrive at its destination.

Methods Used to Add Vectors

Several methods have been developed to add vectors. In this chapter, the graphic method will be explained. The next chapter will explain the component addition method. Either one of these methods will provide fairly accurate results. If a high degree of accuracy is required, an analytical method using geometric and trigonometric functions is required.

Using the Graphic Method

Before attempting to use this method, the following equipment is needed: standard linear (nonlog) graph paper, ruler, protractor, and pencil. The graphic method utilizes a five-step process.

Step 1. Plot the first vector on the rectangular (x-y) axes.

a. Ensure that the same scale is used on both axes.

b. Place the tail (beginning) of the first vector at the origin of the axes as shown in Figure 16.

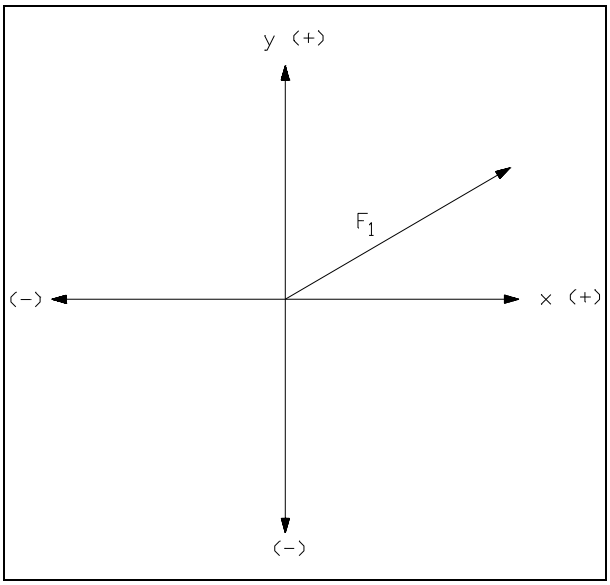

Figure 16 Rectangular Coordinate System

Step 2. Draw the second vector connected to the end of the first vector.

a. Start the tail of the second vector at the head of the first vector.

b. Ensure that the second vector is also drawn to scale.

c. Ensure proper angular orientation of the second vector with respect to the axes of the graph (see Figure 17).

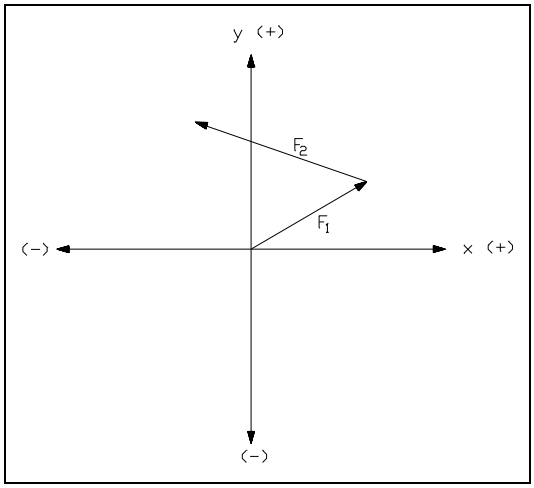

Figure 17 Vector F_2

Step 3. Add other vectors sequentially.

a. Add one vector at a time.

b. Always start the tail of the new vector at the head of the previous vector.

c. Draw all vectors to scale and with proper angular orientation.

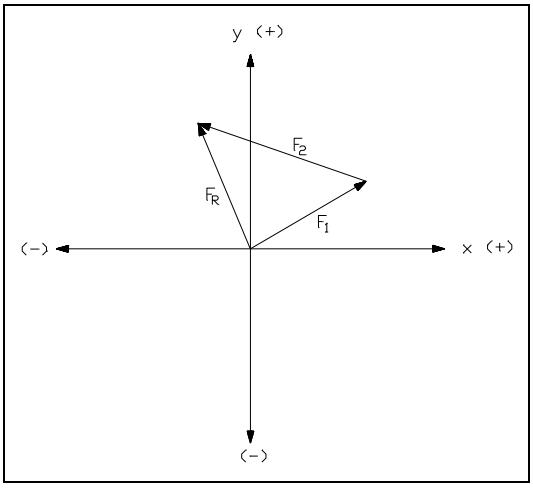

Figure 18 Resultant

Step 4. When all given vectors have been drawn, draw and label a resultant vector, F_R, from the point of origin of the axes to the head of the final vector.

a. The tail of the resultant is the tail of the first vector drawn as shown in Figure 18.

b. The head of the resultant is at the head of the last vector drawn.

Step 5. Determine the magnitude and direction of the resultant.

a. Measure the displacement and angle directly from the graph using a ruler and a protractor.

b. Determine the components of the resultant by projection onto the x- and y-axes.

Example 1: What are the magnitude and direction of the resultant for the following: F_1 = 3 units at 300°, F_2 = 4 units at 60°, and F_3 = 8 units at 180°? The three vectors and their resultant are shown in Figure 19.

Answer: F_R = 4 units at 150°

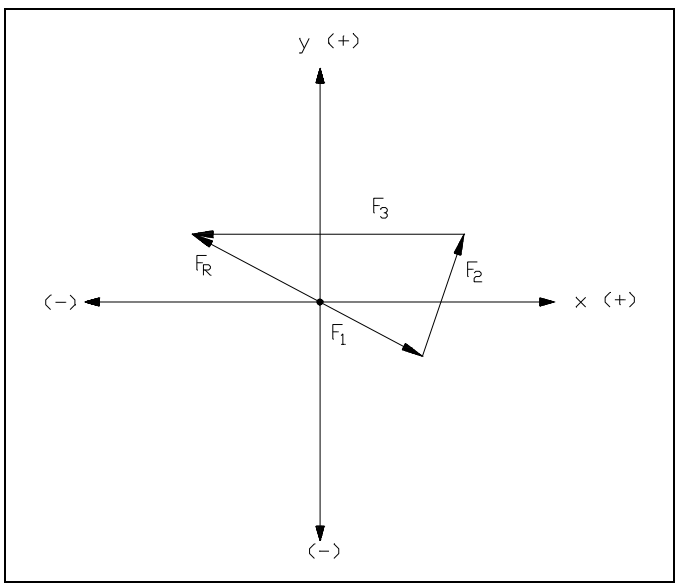

Figure 19 Graphic Addition - Example 1

Example 2: Given X_L = 50 Ohms at 90°, R = 50 Ohms at 0°, and X_c = 50 Ohms at 270°, what is the Resultant Z? (See Figure 20) Note: X_L is inductive reactance, X_c is capacitive reactance and Z is impedance.

Answer: Z = 50 Ohms at 0°

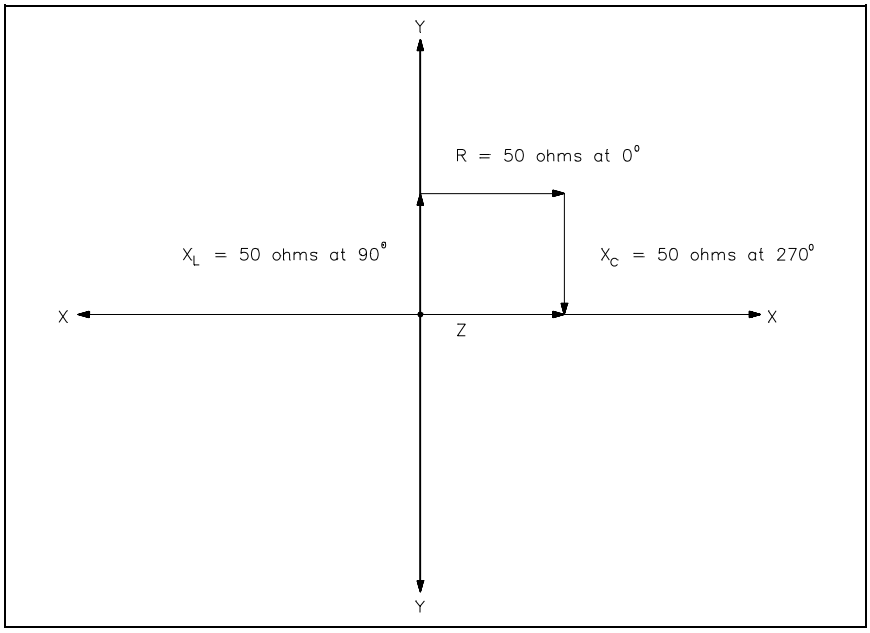

Figure 20 Graphic Addition - Example 2

Summary

The steps of the graphic method of vector addition are summarized below.

Graphic Method Summary

- Draw rectangular coordinates.
- Draw first vector.
- Draw second vector connected to the end (head) of first vector with proper angular orientation.
- Draw remaining vectors, starting at the head of the preceding vector.
- Draw resultant vector from the origin of axes to head of final vector.
- Measure length of resultant.
- Measure angle of resultant vector addition.

COMPONENT ADDITION METHOD

Vector components are added along each axis to determine the magnitude and direction of the resultant.

EO 1.3 ADD vectors using the following methods:
b. Component addition

An Explanation of Components

The component addition method refers to the addition of vector coordinates on a rectangular (x,y) coordinate system. Coordinates, as seen in previous examples, locate a specific point in the system. Relative to vectors, that specific point is the head of the vector. There are two ways to locate that point. The head can be located by counting the units along the x-axis and the units along the y-axis, as illustrated in Figure 21, where the point has coordinates (4,3); i.e., the x component has a magnitude of 4 and the y component has a magnitude of 3.

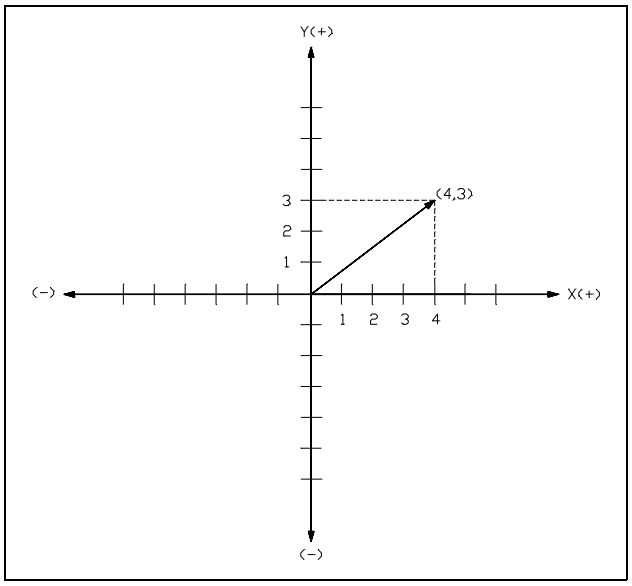

Figure 21 Vector Addition Component Method

The head can also be found by locating a vector of the proper length on the positive side of the x-axis, with its tail at the intersection of the x- and y- axes. Then the vector is rotated a given number of degrees in the counterclockwise direction. In this example, the head of the vector is located five units at 36.9°. Five units is the length of the vector.

Using the Component Addition Method

To add vectors using the component addition method, use the following four step method.

Step 1. Determine x- and y-axes components of all original vectors.

Step 2. Mathematically combine all x-axis components.

Note: When combining, recognize that positive x components at 180° are equivalent to negative x components at 0° (+x at 180° = -x at 0°).

3. Mathematically combine all y-axis components (+y at 270° = -y at 90°).

4. Resulting (x,y) components are the (x,y) components of the resulting vector.

The following examples illustrate vector addition using the component addition method.

Example 1:

Given the following vectors what are the coordinates of the resultant vector, that is, the sum of the vectors?

F_1 = (4,10), F_2 = (-6,4), F_3 = (2,-4), and F_4 = (10,-2)

Step 1. Determine the x- and y-axes components of all four original vectors.

x-axes components = 4, -6, 2, 10
y-axes components = 10, 4, -4, -2

Step 2. Mathematically combine all x-axis components.

$F_x = 4 + (-6) + 2 + 10$
$F_x = 4 - 6 + 2 + 10$
$F_x = 10$

Step 3. Mathematically combine all y-axis components.

$F_y = 10 + 4 + (-4) + (-2)$
$F_y = 10 + 4 - 4 - 2$
$F_y = 8$

Step 4. Express the resultant vector.

The resultant components from the previous additions are the coordinates of the resultant, that is, F_R = (10,8).

Example 2: Determine the resultant, F_R.

Given: F_1 = 30 lbf at 0°, 10 lbf at 90°
F_2 = 50 lbf at 0°, 50 lbf at 90°
F_3 = 45 lbf at 180°, 30 lbf at 90°
F_4 = 15 lbf at 0°, 50 lbf at 270°

Follow the sequence used in the first example. Remember that x at 180º is -x at 0º, and y at 270º is -y at 90º.

$F_x = 30 + 50 + (-45) + 15 = 50$ lbf
$F_y = 10 + 50 + 30 + (-50) = 40$ lbf
$F_R = 50$ lbf at 0º, 40 lbf at 90º

Summary

The sequence of steps used in the component addition method of adding vectors is summarized below.

Component Addition Method Summary

- Determine the x- and y- axes of all original vectors.
- Mathematically combine all x-axis components.
- Mathematically combine all y-axis components.
- The results are the components of the resultant vector.

ANALYTICAL METHOD OF VECTOR ADDITION

Vector components are added to determine the magnitude and direction of the resultant. Calculations using trigonometric functions are the most accurate method for making this determination.

EO 1.3 ADD vectors using the following methods:
c. Analytical

The graphic and components addition methods of obtaining the resultant of several vectors described in the previous chapters can be hard to use and time consuming. In addition, accuracy is a function of the scale used in making the diagram and how carefully the vectors are drawn. The analytical method can be simpler and far more accurate than these previous methods.

Review of Mathematical Functions

In earlier mathematics lessons, the Pythagorean Theorem was used to relate the lengths of the sides of right triangles such as in Figure 22. The Pythagorean Theorem states that in any right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides. This expression may be written as given in Equation 2-4.

$$c^2 = a^2 + b^2 \quad \text{or} \quad c = \sqrt{a^2 + b^2} \tag{2-4}$$

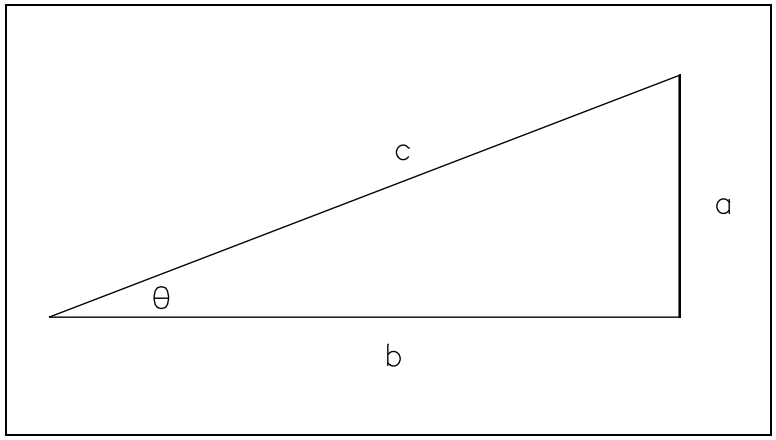

Figure 22 Right Triangle

Also, recall the three trigonometric functions reviewed in an earlier chapter and shown in Figure 23. The cosine will be used to solve for F_x. The sine will be used to solve for F_y. Tangent will normally be used to solve for θ, although sine and cosine may also be used.

$$\text{Sine } \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\text{Cosine } \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\text{Tangent } \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$

Figure 23 Trigonometric Functions

On a rectangular coordinate system, the sine values of θ are positive (+) in quadrants I and II and negative (-) in quadrants III and IV. The cosine values of θ are positive (+) in quadrants I and IV and negative (-) in quadrants II and III. Tangent values are positive (+) in quadrants I and III and negative (-) in quadrants II and IV.

When mathematically solving for tan θ, calculators will specify angles in quadrants I and IV only. Actual angles may be in quadrants II and III. Each problem should be analyzed graphically to report a realistic solution. Quadrant II and III angles may be obtained by adding or subtracting 180° from the value calculated.

Using the Analytical Method

To illustrate this method, consider this example: a man walks 3 miles in one direction, then turns 90° and continues to walk for an additional 4 miles. In what direction and how far is he from his starting point? The first step in solving this problem is to draw a simple sketch as shown in Figure 24.

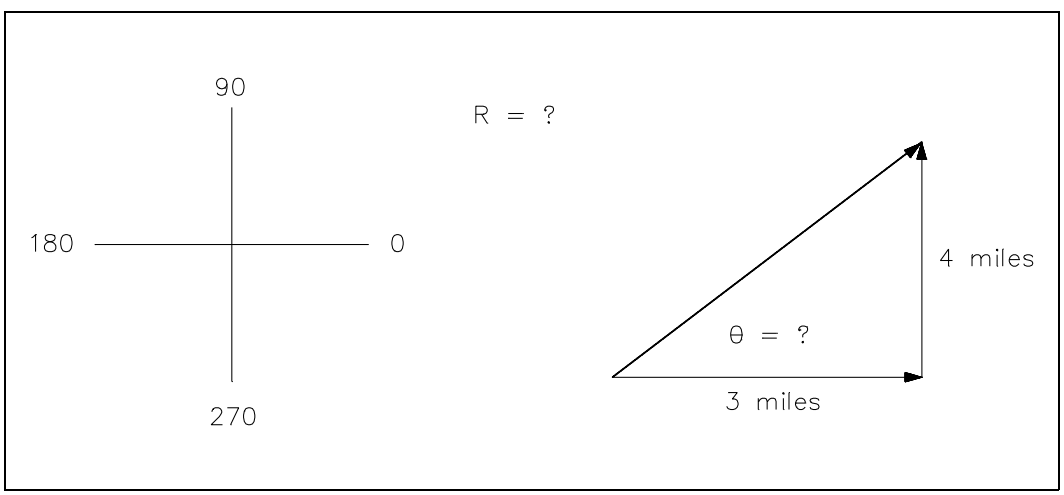

Figure 24 Hypotenuse and Angle

His net displacement is found using equation 2-4.

$$R = \sqrt{a^2 + b^2}$$

$$R = \sqrt{3^2 + 4^2}$$

$$R = \sqrt{25}$$

$$R = 5 \text{ miles}$$

His direction (angle of displacement) is found using the tangent function.

$\tan \theta$ = opposite/adjacent
$\tan \theta = a/b$
$\tan \theta = 4/3$
$\tan \theta = 1.33$
$\theta = \tan^{-1} 1.33$
$\theta = 53^o$

Therefore, his new location is 5 miles at 53° from his starting point.

By carrying this approach a step further, a model has been developed for finding the resultant of several vectors. For the purpose of developing the model, consider three forces (F_1, F_2, and F_3) acting on an object as shown in Figure 25. The goal is to find the resultant force (F_R).

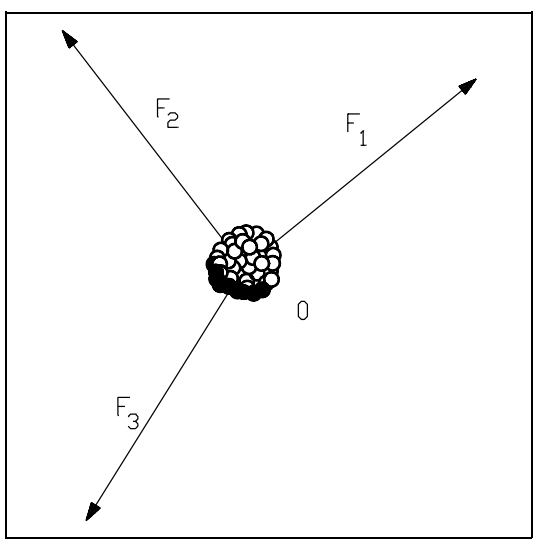

Figure 25 Example Model 1

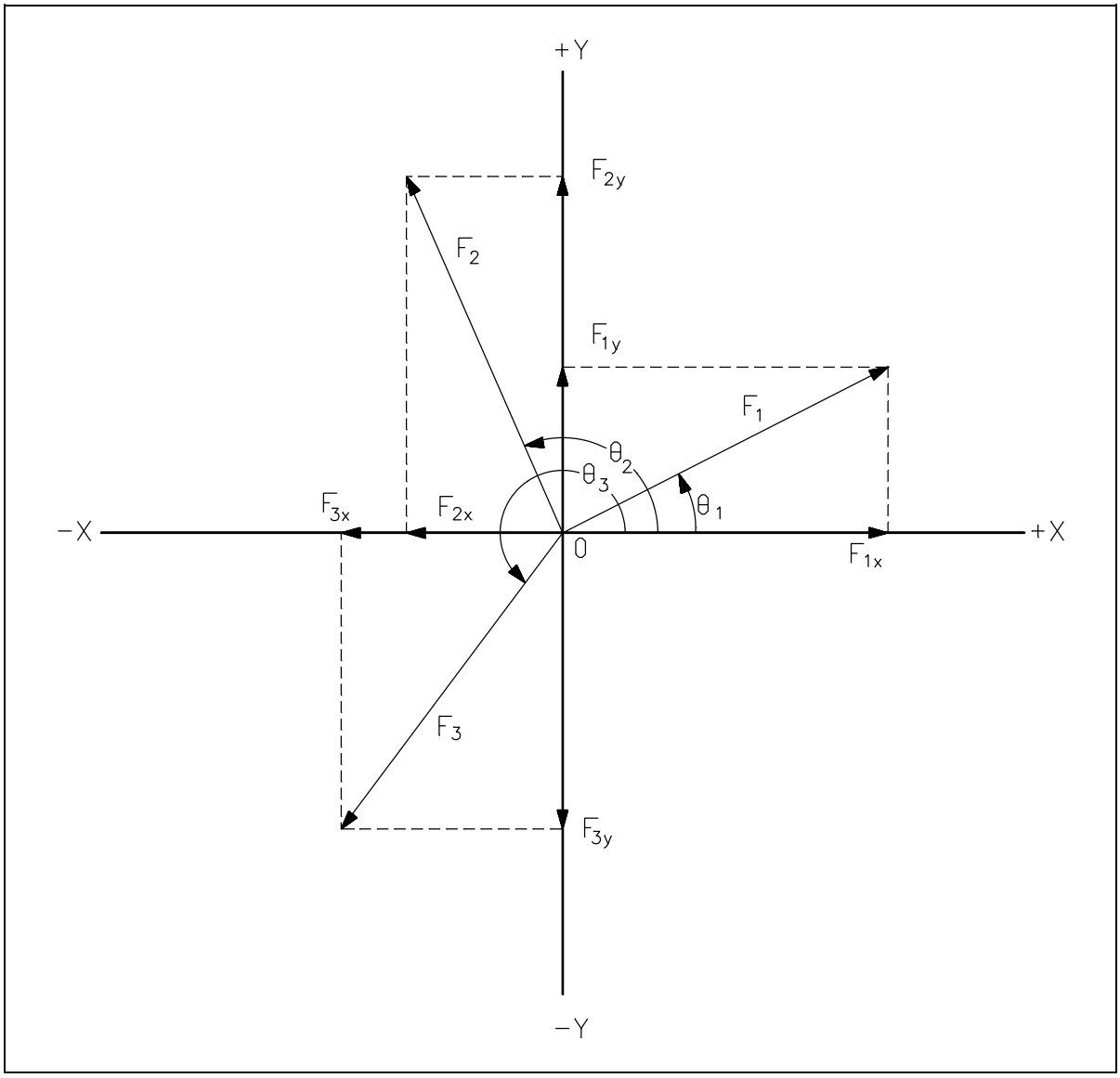

Figure 26 Example Model 2

Step 1: Draw x and y coordinates and the three forces from the point of origin or the center of the object, as shown in Figure 26. Component vectors and angles have been added to the drawing to aid in the discussion.

Step 2: Resolve each vector into its rectangular components.

Vector	Angle	x component	y component
F_1	θ_1	$F_{1x} = F_1\cos\theta_1$	$F_{1y} = F_1\sin\theta_1$
F_2	θ_2	$F_{2x} = F_2\cos\theta_2$	$F_{2y} = F_2\sin\theta_2$
F_3	θ_3	$F_{3x} = F_3\cos\theta_3$	$F_{3y} = F_3\sin\theta_3$

Step 3: Sum the x and y components.

$$F_{Rx} = \Sigma F_x = F_{1x} + F_{2x} + F_{3x}$$

$$F_{Ry} = \Sigma F_y = F_{1y} + F_{2y} + F_{3y}$$

Where "Σ" means summation

Step 4: Calculate the magnitude of F_R.

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

Step 5: Calculate the angle of displacement.

$$\tan\theta = \frac{F_{Ry}}{F_{Rx}}$$

$$\theta = \tan^{-1}\frac{F_{Ry}}{F_{Rx}}$$

Here is an example using this model. Follow it through step by step.

Example: Given three forces acting on an object, determine the magnitude and direction of the resultant force F_R.

F_1 = 90 lbf at 39°

F_2 = 50 lbf at 120°

F_3 = 125 lbf at 250°

Step 1: First draw x and y coordinate axes on a sheet of paper. Then, draw F_1, F_2, and F_3 from the point of origin. It is not necessary to be totally accurate in placing the vectors in the drawing. The approximate location in the right quadrant is all that is necessary. Label the drawing as in the model (Figure 26).

Step 2: Resolve each force into its rectangular coordinates.

Force	Magnitude	Angle	x component	y component
F_1	90 lbf	39°	F_{1x} = 90 cos 39° F_{1x} = (90) (.777) F_{1x} = 69.9 lbf	F_{1y} = 90 sin 39° F_{1y} = (90) (.629) F_{1y} = 56.6 lbf
F_2	50 lbf	120°	F_{2x} = 50 cos 120° F_{2x} = (50) (-.5) F_{2x} = -25 lbf	F_{2y} = 50 sin 120° F_{2y} = (50) (.866) F_{2y} = 43.3 lbf
F_3	125 lbf	250°	F_{3x} = 125 cos 250° F_{3x} = (125) (-.342) F_{3x} = -42.8 lbf	F_{3y} = 125 sin 250° F_{3y} = (125) (-.94) F_{3y} = -117.5 lbf

Step 3: Sum the x and y components.

$F_{Rx} = F_{1x} + F_{2x} + F_{3x}$

$F_{Rx} = 69.9 \text{ lbf} + (-25 \text{ lbf}) + (-42.8 \text{ lbf})$

$F_{Rx} = 2.1 \text{ lbf}$

$F_{Ry} = F_{1y} + F_{2y} + F_{3y}$

$F_{Ry} = 56.6 \text{ lbf} + 43.3 \text{ lbf} + (-117.5 \text{ lbf})$

$F_{Ry} = -17.6 \text{ lbf}$

Step 4: Calculate the magnitude of F_R.

$F_R = \sqrt{F_x^2 + F_y^2}$

$F_R = \sqrt{(2.1)^2 + (-17.6)^2}$

$F_R = \sqrt{314.2}$

$F_R = 17.7 \text{ lbf}$

Step 5: Calculate the angle of displacement.

$\tan \theta = F_{Ry}/F_{Rx}$

$\tan \theta = -17.6/2.1$

$\tan \theta = -8.381$

$\theta = \tan^{-1} (-8.381)$

$\theta = -83.2^o$

Therefore, $F_R = 17.7$ lbf at -83.2^o or 276.8^o.

Note: A negative angle means a clockwise rotation from the zero axis.

It is left to the student to try the previous example using the other methods of vector addition described in earlier chapters.

Summary

The steps followed when using the analytical method to find the resultant of several vectors are summarized below.

Analytical Method of Adding Vectors Summary

- Draw x and y coordinate axes.
- Draw component vectors from point of origin.
- Resolve each vector into rectangular components.
- Sum x and y components.
- Calculate magnitude of F_R.
- Calculate angle of displacement.

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CLASSICAL PHYSICS
Module 3
Force and Motion

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REFERENCES

- Beiser, Arthur, Applied Physics Schaums Outline Series, McGraw-Hill Book Company, 1976.
- Murray, Raymond and Cobb, Grover, Physics Concepts and Consequences, Prentice-Hall, Inc., 1970.
- Alexander, Joseph, et al., Physics for Engineering Technology, John Wiley and Sons, 1978.
- Sears, Francis and Zemansky, Mark, University Physics, 3rd edition, Addison-Wesley Publishing Co.
- Science and Fundamental Engineering (SAFE), Classical Physics, "Measure" Windsor, CT: Combustion Engineering, Inc., 1988.
- Academic Program for Nuclear Power Plant Personnel, Volume II, General Physics Corporation, Library of Congress Card #A 397747, April 1982.

TERMINAL OBJECTIVE

1.0 **APPLY** Newton's laws of motion to a body.

ENABLING OBJECTIVES

1.1 **STATE** Newton's first law of motion.

1.2 **STATE** Newton's second law of motion.

1.3 **STATE** Newton's third law of motion.

1.4 **STATE** Newton's law of universal gravitation.

1.5 **DEFINE** momentum.

1.6 **EXPLAIN** the conservation of momentum.

1.7 Using the conservation of momentum, **CALCULATE** the velocity for an object (or objects) following a collision of two objects.

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NEWTON'S LAWS OF MOTION

The study of Newton's laws of motion allows us to understand and accurately describe the motion of objects and the forces that act on those objects.

EO 1.1 STATE Newton's first law of motion.

EO 1.2 STATE Newton's second law of motion.

EO 1.3 STATE Newton's third law of motion.

EO 1.4 STATE Newton's law of universal gravitation.

The basis for modern mechanics was developed in the seventeenth century by Sir Isaac Newton. From his studies of objects in motion, he formulated three fundamental laws.

Newton's first law of motion states "an object remains at rest (if originally at rest) or moves in a straight line with constant velocity if the net force on it is zero."

Newton's second law states "the acceleration of a body is proportional to the net (i.e., sum or resultant) force acting on it and in the direction of that net force." This law establishes the relationship between force, mass, and acceleration and can be written mathematically as shown in Equation 3-1.

$$F = ma \tag{3-1}$$

where:

F = force (Newton = 1 Kg-m/sec^2, or lbf)
m = mass (Kg or lbm)
a = acceleration (m/sec^2 or ft/sec^2)

This law is used to define force units and is one of the most important laws in physics. Also, Newton's first law is actually a consequence of this second law, since there is no acceleration when the force is zero, and the object is either at rest or moving with a constant velocity. Equation 3-1 can be used to calculate an objects weight at the surface of the earth. In this special case, F is the force, or weight, caused by the gravitational acceleration of the earth acting on the mass, m, of the object. When dealing with this type of problem, we designate the acceleration, g, which equals 9.8m/sec^2 or 32.17 ft/sec^2 (g is called gravitational acceleration constant). Thus, equation 3-1 becomes F = mg for this case.

Newton's third law of motion states "if a body exerts a force on a second body, the second body exerts an equal and opposite force on the first." This law has also been stated as, "for every action there is an equal and opposite reaction."

The third law is basic to the understanding of force. It states that forces always occur in pairs of equal and opposite forces. Thus, the downward force exerted on a desk by a pencil is accompanied by an upward force of equal magnitude exerted on the pencil by the desk. This principle holds for all forces, variable or constant, regardless of their source.

One additional law attributed to Newton concerns mutual attractive forces between two bodies. It is known as the universal law of gravitation and is stated as follows.

> "Each and every mass in the universe exerts a mutual, attractive gravitational force on every other mass in the universe. For any two masses, the force is directly proportional to the product of the two masses and is inversely proportional to the square of the distance between them."

Newton expressed the universal law of gravitation using Equation 3-2.

$$F = G\frac{m_1 m_2}{r^2} \tag{3-2}$$

where:

F	=	force of attraction (Newton = 1Kg-m/sec^2 or lbf)
G	=	universal constant of gravitation (6.673×10^{-11} m^3/kg-sec^2 or 3.44×10^{-8} $\frac{\text{lbm} - \text{ft}^2}{\text{slug}^2}$)
m_1	=	mass of the first object (Kg or lbm)
m_2	=	mass of the second object (Kg or lbm)
r	=	distance between the centers of the two objects (m or ft)

Using this universal law of gravitation, we can determine the value of g (gravitational acceleration constant), at the surface of the earth. We already know this value to be 9.8 m/sec^2 (or 32.17 ft/sec^2), but it can be calculated using Equation 3-2.

Calculation:

> First, we will assume that the earth is much larger than the object and that the object resides on the surface of the earth; therefore, the value of r will be equal to the radius of the earth. Second, we must understand that the force of attraction (F) in Equation 3-2 for the object is equal to the object's weight (F) as described in Equation 3.1. Setting these two equations equal to each other yields the following.

$$F = G\frac{M_e m_1}{r^2} = m_1 a$$

where:

M_e = mass of the earth (5.95×10^{24} kg)
m_1 = mass of the object
r = radius of the earth (6.367×10^6 m)

The mass (m_1) of the object cancels, and the value of (g) can be determined as follows since a=g by substituting (g) for (a) in the previous equation.

$$g = G\frac{M_e}{r^2}$$

$$g = \left(6.673 \times 10^{-11} \frac{m^3}{kg\ sec^2}\right)\left(\frac{5.95 \times 10^{24}\ kg}{(6.367 \times 10^6\ m)^2}\right)$$

$$g = 9.8 \frac{m}{sec^2}$$

If the object is a significant distance from the earth, we can demonstrate that (g) is not a constant value but varies with the distance (altitude) from the earth. If the object is at an altitude of 30 km (18.63 mi), then the value of (g) is as follows:

$$r = 30000\ m + 6.367 \times 10^6\ m = 6.397 \times 10^6\ m$$

$$g = \left(6.673 \times 10^{-11} \frac{m^3}{kg\ sec^2}\right)\left(\frac{5.95 \times 10^{24}\ kg}{(6.397 \times 10^6\ m)^2}\right)$$

$$g = 9.7 \frac{m}{sec^2}$$

As you can see, a height of 30 km only changes (g) from 9.8 m/sec^2 to 9.7 m/sec^2. There will be an even smaller change for objects closer to the earth. Therefore, (g) is normally considered a constant value since most calculations involve objects close to the surface of the earth.

Summary

Newton's laws of motion are summarized below.

Newton's Laws of Motion Summary

Newton's First Law of Motion

An object remains at rest (if originally at rest) or moves in a straight line with constant velocity if the net force on it is zero.

Newton's Second Law of Motion

A particle with a force acting on it has an acceleration proportional to the magnitude of the force and in the direction of that force.

Newton's Third Law of Motion

The forces of action and reaction between interacting bodies are equal in magnitude and opposite in direction.

- or -

For every action there is an equal and opposite reaction.

Newton's Universal Law of Gravitation

Each and every mass in the universe exerts a mutual, attractive gravitational force on every other mass in the universe. For any two masses, the force is directly proportional to the product of the two masses and is inversely proportional to the square of the distance between them.

MOMENTUM PRINCIPLES

Momentum is a measure of the motion of a moving body. An understanding of momentum and the conservation of momentum provides essential tools in solving physics problems.

EO 1.5 **DEFINE momentum.**

EO 1.6 **EXPLAIN the conservation of momentum.**

EO 1.7 **Using the conservation of momentum, CALCULATE the velocity for an object (or objects) following a collision of two objects.**

Momentum

Momentum is a basic and widely applicable concept of physics. It is, in a sense, the measure of the motion of a moving body. It is the result of the product of the body's mass and the velocity at which it is moving. Therefore, momentum can be defined using Equation 3-3.

$$P = mv \tag{3-3}$$

where:

P = momentum of the object (Kg-m/sec or ft-lbm/sec)
m = mass of the object (Kg or lbm)
v = velocity of the object (m/sec or ft/sec)

Momentum is a vector quantity since it results from the velocity of the object. If different momentum quantities are to be added vectorially, the direction of each momentum must be taken into account. However, to simplify the understanding of momentum, only straight line motions will be considered.

Example:

Calculate the momentum for a 16 lbm bowling ball rolling down a lane at 22 ft/sec.

Solution:

$$P = mv$$

$$P = (16\ \text{lbm})\left(22\ \frac{\text{ft}}{\text{sec}}\right)$$

$$P = 352\frac{\text{ft-lbm}}{\text{sec}}$$

Force and Momentum

There is a direct relationship between force and momentum. The rate at which momentum changes with time is equal to the net force applied to an object. This relationship comes directly from Newton's second law of motion, *F = ma*. This is a special case of Newton's second law for a constant force which gives rise to a constant acceleration. The linking fact is that acceleration is the rate at which velocity changes with time. Therefore, we can determine the following:

We know that, $F = ma$

and since, $a = \dfrac{(v - v_o)}{(t - t_o)}$

then, $F = m\left(\dfrac{v - v_o}{t - t_o}\right)$ (3-4)

which can also be written, $F = \dfrac{mv - mv_o}{t - t_o}$ (3-5)

Substituting P for mv and P_o for mv_o, $F = \dfrac{P - P_o}{t - t_o}$

or $F = \dfrac{\Delta P}{\Delta t}$ (3-6)

From Equation 3-6, we can determine that force (F) is equal to the change in momentum per time.

Example:

The velocity of a rocket must be increased by 110 ft/sec to achieve proper orbit around the earth. If the rocket has a mass of 5 tons and it takes 9 sec. to reach orbit, calculate the required thrust (force) to achieve this orbit.

Solution:

Even though the initial velocity (v_o) and final velocity (v) are unknown, we do know the change in velocity ($v-v_o$), which is 110 ft/sec. Therefore, using Equation 3-4 we can find the solution.

$$F = m\left(\frac{v - v_o}{t - t_o}\right)$$

$$F = 10000 \text{ lbm}\left(\frac{110 \text{ ft/sec}}{9 \text{ sec}}\right)$$

$$F = 1.2 \times 10^5 \ \frac{\text{ft-lbm}}{\text{sec}^2}$$

Conservation of Momentum

One of the most useful properties of momentum is that it is conserved. This means that if no net external force acts upon an object, the momentum of the object remains constant. Using Equation 3-6, we can see that if force (F) is equal to zero, then $\Delta P = 0$. It is most important for collisions, explosions, etc., where the external force is negligible, and states that the momentum before the event (collision, explosion) equals the momentum following the event.

The conservation of momentum applies when a bullet is fired from a gun. Prior to firing the gun, both the gun and the bullet are at rest (i.e., V_G and V_B are zero), and therefore the total momentum is zero. This can be written as follows:

$$m_G v_G + m_B v_B = 0$$

or

$$m_G v_G = -m_B v_B$$

When the gun is fired, the momentum of the recoiling gun is equal and opposite to the momentum of the bullet. That is, the momentum of the bullet ($m_B v_B$) is equal to the momentum of the gun ($m_G v_G$), but of opposite direction.

The development of the law of conservation of momentum does not consider whether the collision is elastic or inelastic. In an elastic collision, both momentum and kinetic energy (i.e., energy due to an objects velocity) are conserved. A common example of an elastic collision is the head-on collision of two billiard balls of equal mass. In an inelastic collision, momentum is conserved, but system kinetic energy is not conserved. An example of an inelastic collission is the head-on collision of two automobiles where part of the initial kinetic energy is lost as the metal crumples during the impact. The concept of kinetic energy will be discussed further in Module 5 of this course.

The law of conservation of momentum can be mathematically expressed in several different ways. In general, it can be stated that the sum of a system's initial momentum is equal to the sum of a system's final momentum.

$$\sum P_{initial} = \sum P_{final} \tag{3-7}$$

In the case where a collision of two objects occurs, the conservation of momentum can be stated as follows.

$$P_{1\ initial} + P_{2\ initial} = P_{1\ final} + P_{2\ final} \tag{3-8}$$

or

$$(m_1v_1)_{initial} + (m_2v_2)_{initial} = (m_1v_1)_{final} + (m_2v_2)_{final} \tag{3-9}$$

In the case where two bodies collide and have identical final velocities, equation 3-10 applies.

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v_f \tag{3-10}$$

For example, consider two railroad cars rolling on a level, frictionless track (see Figure 1). The cars collide, become coupled, and roll together at a final velocity (v_f). The momentum before and after the collision is expressed with Equation 3-10.

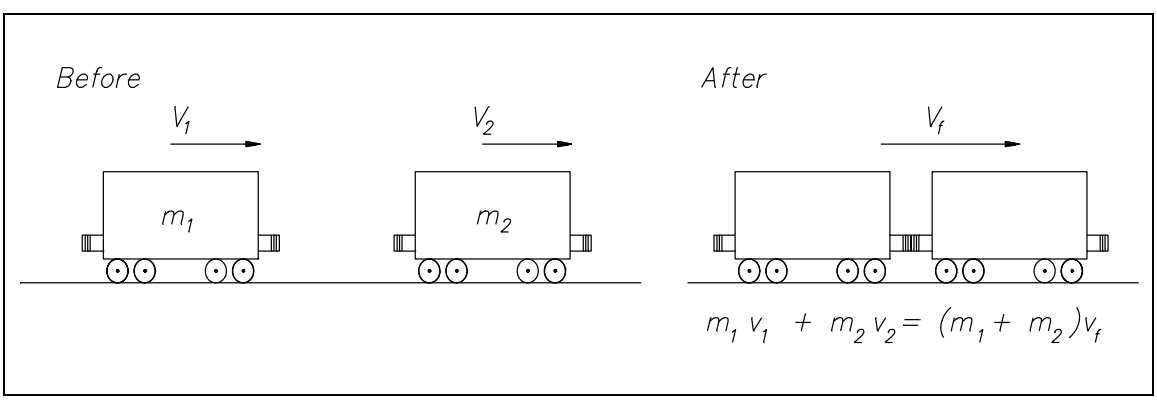

Figure 1 Momentum

If the initial velocities of the two objects (v_1 and v_2) are known, then the final velocity (v_f) can be calculated by rearranging Equation 3-10 into Equation 3-11.

$$v_f = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \tag{3-11}$$

Example:

Consider that the railroad cars in Figure 1 have masses of m_1 = 2300 lbm and m_2 = 2800 lbm. The first car (m_1) is moving at a velocity of 29 ft/sec and the second car (m_2) is moving at a velocity of 11 ft/sec. The first car overtakes the second car and couples with it. Calculate the final velocity of the two cars.

Solution:

The final velocity (v_f) can be easily calculated using Equation 3-8.

$$v_f = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$v_f = \frac{(2300\ \text{lbm})(29\ \text{ft/sec}) + (2800\ \text{lbm})(11\ \text{ft/sec})}{2300\text{lbm} + 2800\ \text{lbm}}$$

$$v_f = 19.1\ \text{ft/sec}$$

Summary

The concepts of momentum and the conservation of momentum are summarized below.

Momentum Summary

Momentum is the measure of the motion of a moving body. It is the result of the product of the body's mass and the velocity at which it is moving. Therefore, momentum can be defined as:

$$P = mv.$$

The conservation of momentum states that if no net external force acts upon a system, the momentum of the system remains constant. If force (F) is equal to zero, then $\Delta P = 0$.

The momentum before and after a collision can be calculated using the following equation.

$$(m_1v_1)_{initial} + (m_2v_2)_{initial} = (m_1v_1)_{final} + (m_2v_2)_{final}$$

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Fundamentals Handbook

CLASSICAL PHYSICS
Module 4
Application of Newton's Laws

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- Beiser, Arthur, Applied Physics Schaums Outline Series, McGraw-Hill Book Company, 1976.
- Murray, Raymond and Cobb, Grover, Physics Concepts and Consequences, Prentice-Hall, Inc., 1970.
- Alexander, Joseph, et al., Physics for Engineering Technology, John Wiley and Sons, 1978.
- Sears, Francis and Zemansky, Mark, University Physics, 3rd edition, Addison-Wesley Publishing Co.
- Science and Fundamental Engineering (SAFE), Classical Physics, "Measure" Windsor, CT: Combustion Engineering, Inc., 1988.
- Academic Program for Nuclear Power Plant Personnel, Volume II, General Physics Corporation, Library of Congress Card #A 397747, April 1982.

TERMINAL OBJECTIVE

1.0 From memory, **APPLY** the principles of force to stationary or moving bodies.

ENABLING OBJECTIVES

1.1 **DEFINE** the following:
- a. Force
- b. Weight

1.2 **STATE** the purpose of a free-body diagram.

1.3 Given all necessary information, **CONSTRUCT** a free-body diagram.

1.4 **STATE** the conditions necessary for a body to be in force equilibrium.

1.5 **DEFINE** the following:
- a. Net force
- b. Equilibrant

1.6 **DEFINE** the following:
- a. Tensile force
- b. Compressive force
- c. Frictional force

1.7 **EXPLAIN** the difference between a static-friction force and a kinetic-friction force.

1.8 **STATE** two factors that affect the magnitude of the friction force.

1.9 **EXPLAIN** the difference between centripetal force and centrifugal force.

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FORCE AND WEIGHT

Force can be thought of simply as a push or pull, but is more clearly defined as any action on a body that tends to change the velocity of the body. Weight is a force exerted on an object due to the object's position in a gravitational field.

EO 1.1 DEFINE the following:
a. Force
b. Weight

Introduction

In the study of forces, the student must make valid assumptions called for in the formulation of real problems. The ability to understand and make use of the correct assumptions in the formulation and solution of engineering problems is certainly one of the most important abilities of a successful operator. One of the objectives of this manual is to provide an opportunity to develop this ability through the study of the fundamentals and the analysis of practical problems.

An effective method of attack on all engineering problems is essential. The development of good habits in formulating problems and in representing their solutions will prove to be a valuable asset. Each solution should proceed with a logical sequence of steps from hypothesis to conclusion, and its representation should include a clear statement of the following parts, each clearly defined: a) given data, b) results desired, c) necessary diagrams, d) calculations, and e) answers and conclusions. Many problems become clear and straightforward once they are begun with a logical and disciplined method of attack.

In addition, it is important to incorporate a series of checks on the calculations at intermediate points in the solution. Also, all work must be neat and orderly.

The subject of classical physics is based on surprisingly few fundamental concepts and involves mainly the application of these basic relations to a variety of situations. Newton's laws of motion are some of the fundamental concepts used in the study of force and weight.

Force

Force is defined as a vector quantity that tends to produce an acceleration of a body in the direction of its application. Changing the body's velocity causes the body to accelerate. Therefore, force can be mathematically defined as given by Newton's second law of motion (Equation 4-1).

$$F = ma \tag{4-1}$$

where:

F = force on object (Newton or lbf)
m = mass of object (Kg or lbm)
a = acceleration of object (m/sec^2 or ft/sec^2)

Force is characterized by its point of application, its magnitude, and its direction. A force that is actually distributed over a small area of the body upon which it acts may be considered a concentrated force if the dimensions of the area involved are small compared with other pertinent dimensions.

Two or more forces may act upon an object without affecting its state of motion. For example, a book resting upon a table has a downward force acting on it caused by gravity and an upward force exerted on it from the table top. These two forces cancel and the net force of the book is zero. This fact can be verified by observing that no change in the state of motion has occurred.

Weight

Weight is a special application of the concept of force. It is defined as the force exerted on an object by the gravitational field of the earth, or more specifically the pull of the earth on the body.

$$W = \frac{mg}{g_c} \tag{4-2}$$

where:

W = weight (lbf)
m = mass (lbm) of the object
g = the local acceleration of gravity (32.17 ft/sec^2)
g_c = a conversion constant employed to facilitate the use of Newton's second law of motion with the English system of units and is equal to 32.17 $ft\text{-}lbm/lbf\text{-}sec^2$

Note that g_c has the same numerical value as the acceleration of gravity at sea level.

The mass of a body is the same wherever the body is, whether on the moon or on the earth. The weight of a body, however, depends upon the local acceleration of gravity. Thus, the weight of an object is less on the moon than on the earth because the local acceleration of gravity is less on the moon than on the earth.

Example:

Calculate the weight of a person with a mass of 185 lbm.

$$W = \frac{mg}{g_c}$$

$$= \frac{(185\ \text{lbm})\left(32.17\ \frac{\text{ft}}{\text{sec}^2}\right)}{32.17\ \frac{\text{ft-lbm}}{\text{lbf-sec}^2}}$$

$$= 185\ \text{lbf}$$

Example:

Calculate the weight of a person with a mass of 185 lbm on the moon. Gravity on the moon is 5.36 ft/sec^2.

$$W = \frac{mg}{g_c}$$

$$= \frac{(185\ \text{lbm})\left(5.36\ \frac{\text{ft}}{\text{sec}^2}\right)}{32.17\ \frac{\text{ft-lbm}}{\text{lbf-sec}^2}}$$

$$= 28.19\ \text{lbf}$$

With the idea of mass and weight understood, especially their differences, the concept of gravitational force is more easily explained. Any object that is dropped will accelerate as it falls, even though it is not in physical contact with any other body. To explain this, the idea of gravitational force was developed, resulting in the concept that one body, such as the earth, exerts a force on another body, even though they are far apart. The gravitational attraction of two objects depends upon the mass of each and the distance between them. This concept is known as Newton's law of gravitation, which was introduced in an earlier chapter.

Summary

The important concepts of force and weight are summarized below.

Force and Weight Summary

- Force is a vector quantity that tends to produce an acceleration of a body in the direction of its application.

  or

  $F = ma$

- Weight is the force exerted on an object due to gravity. (On the earth it is the gravitational pull of the earth on the body.)

  $W = mg/g_c$

FREE-BODY DIAGRAMS

In studying the effect of forces on a body it is necessary to isolate the body and determine all forces acting upon it. This method of using a free-body diagram is essential in understanding basic and complex force problems.

EO 1.2 **STATE the purpose of a free-body diagram.**

EO 1.3 **Given all necessary information, CONSTRUCT a free-body diagram.**

In solving a problem involving forces it is essential that Newton's laws are carefully fixed in mind and that these principles are applied literally and exactly. In applying these principles it is essential that the body be isolated from all other bodies so that a complete and accurate account of all forces which act on this body may be considered. The diagram of such an isolated body with the representation of all external forces acting on it is called a *Free-Body Diagram*. It has long been established that the free-body-diagram method is the key to the understanding of engineering problems. This is because the isolation of a body is the tool that clearly separates cause and effect and focuses our attention to the literal application of a principle.

Example:

Consider the book resting on the table in Figure 1. Although the book is stationary, two forces are acting on the book to keep it stationary. One is the weight (W) of the book exerting a force down on the table. The other is the force exerted up by the table to hold the book in place. This force is known as the normal force (N) and is equal to the weight of the book. A normal force is defined as any perpendicular force with which any two surfaces are pressed against each other. The free-body diagram for this situation, illustrated on the right side in Figure 1, isolates the book and presents the forces acting on the object.

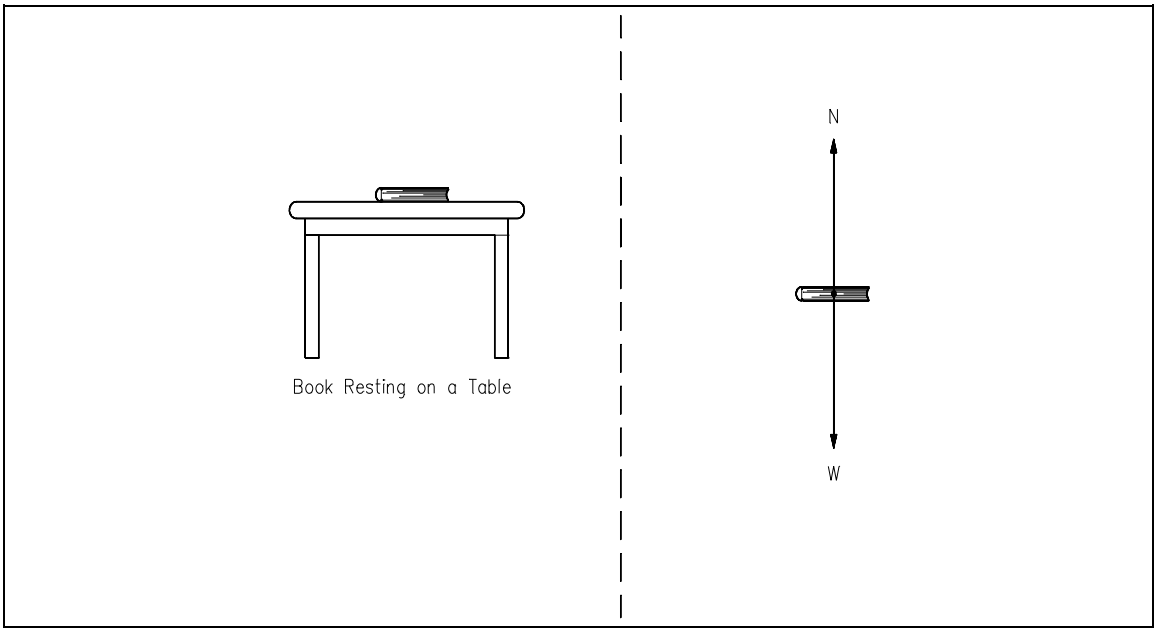

Figure 1 Book on a Table

Constructing a Free-Body Diagram

In constructing a free-body diagram the following steps are usually followed.

Step 1. Determine which body or combination of bodies is to be isolated. The body chosen will usually involve one or more of the desired unknown quantities.

Step 2. Next, isolate the body or combination of bodies chosen with a diagram that represents its complete external boundaries.

Step 3. Represent all forces that act on the isolated body as applied by the removed contacting and attracting bodies in their proper positions in the diagram of the isolated body. Do not show the forces that the object exerts on anything else, since these forces do not affect the object itself.

Step 4. Indicate the choice of coordinate axes directly on the diagram. Pertinent dimensions may also be represented for convenience. Note, however, that the free-body diagram serves the purpose of focusing accurate attention on the action of the external forces; therefore, the diagram should not be cluttered with excessive information. Force arrows should be clearly distinguished from other arrows to avoid confusion. For this purpose colored pencils may be used.

When these steps are completed a correct free-body diagram will result, and the student can apply the appropriate equations to the diagram to find the proper solution.

Example:

The car in Figure 2 is being towed by a force of some magnitude. Construct a free-body diagram showing all the forces acting on the car.

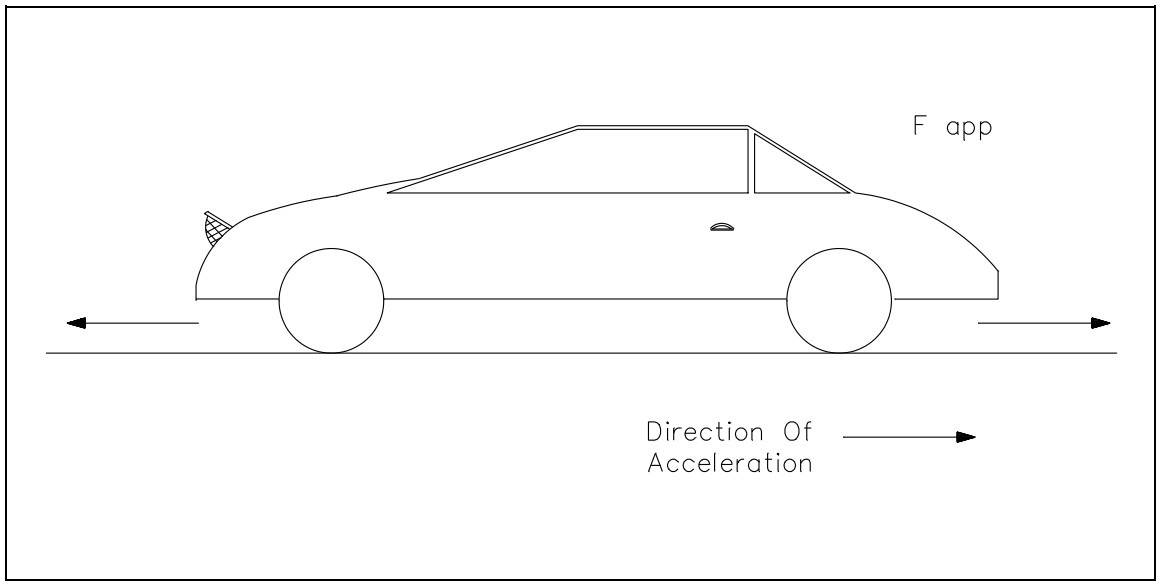

Figure 2 Car

Solution:

Following the steps to construct a free-body diagram (shown in Figure 3), the object (the car) is chosen and isolated. All the forces acting on the car are represented with proper coordinate axes. Those forces are:

F_{app}	-	The force applied to tow the car
F_K	-	The frictional force that opposes the applied force due to the weight of the car and the nature of the surfaces (the car's tires and the road)
W	-	The weight of the car
N	-	The normal force exerted by the road on the car

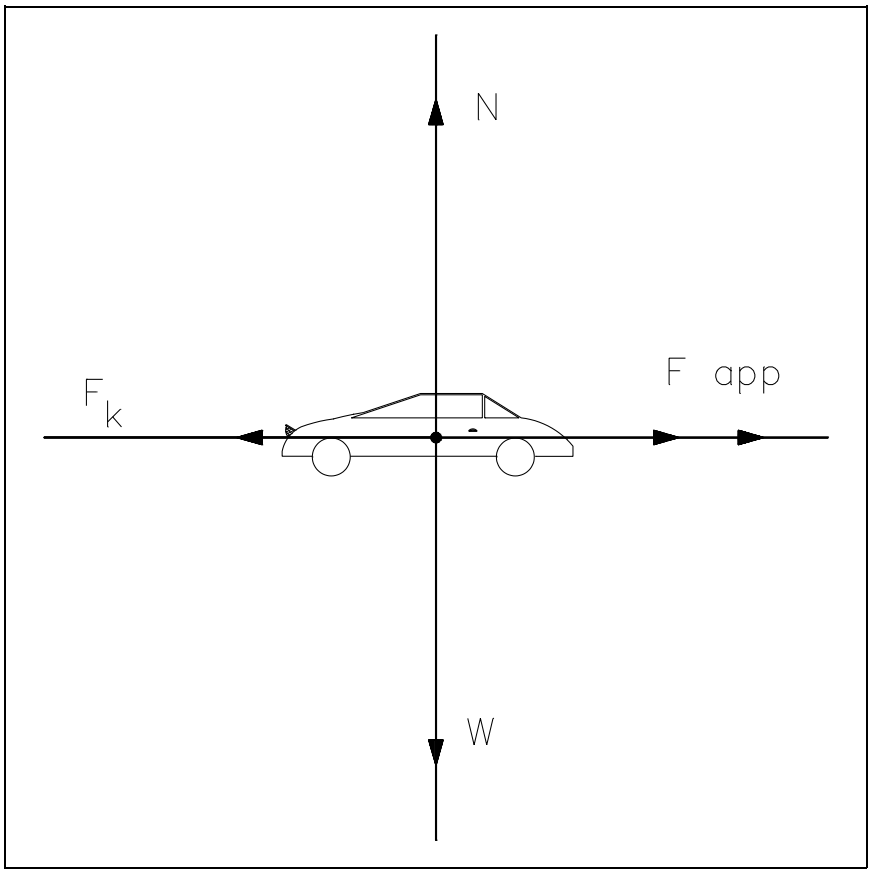

Figure 3 Free-Body Diagram

The frictional force (F_K) is a force that opposes the direction of motion. This force is explained in more detail in the chapter on types of forces.

To solve this practical problem, the student would assign values for each force as determined by data given in the problem. After assigning a sign convention (e.g., + for forces upward and to the right, - for forces downward and to the left), the student would sum all forces to find the net force acting on the body. Using this net force information and appropriate equations, the student could solve for the requested unknowns. A variation would be to have the student find an unknown force acting on the body given sufficient information about the other forces acting on the body. The student will learn to solve specific examples using free-body diagrams in a later chapter.

Some advanced free-body diagrams for various types of systems are shown in Figure 4.

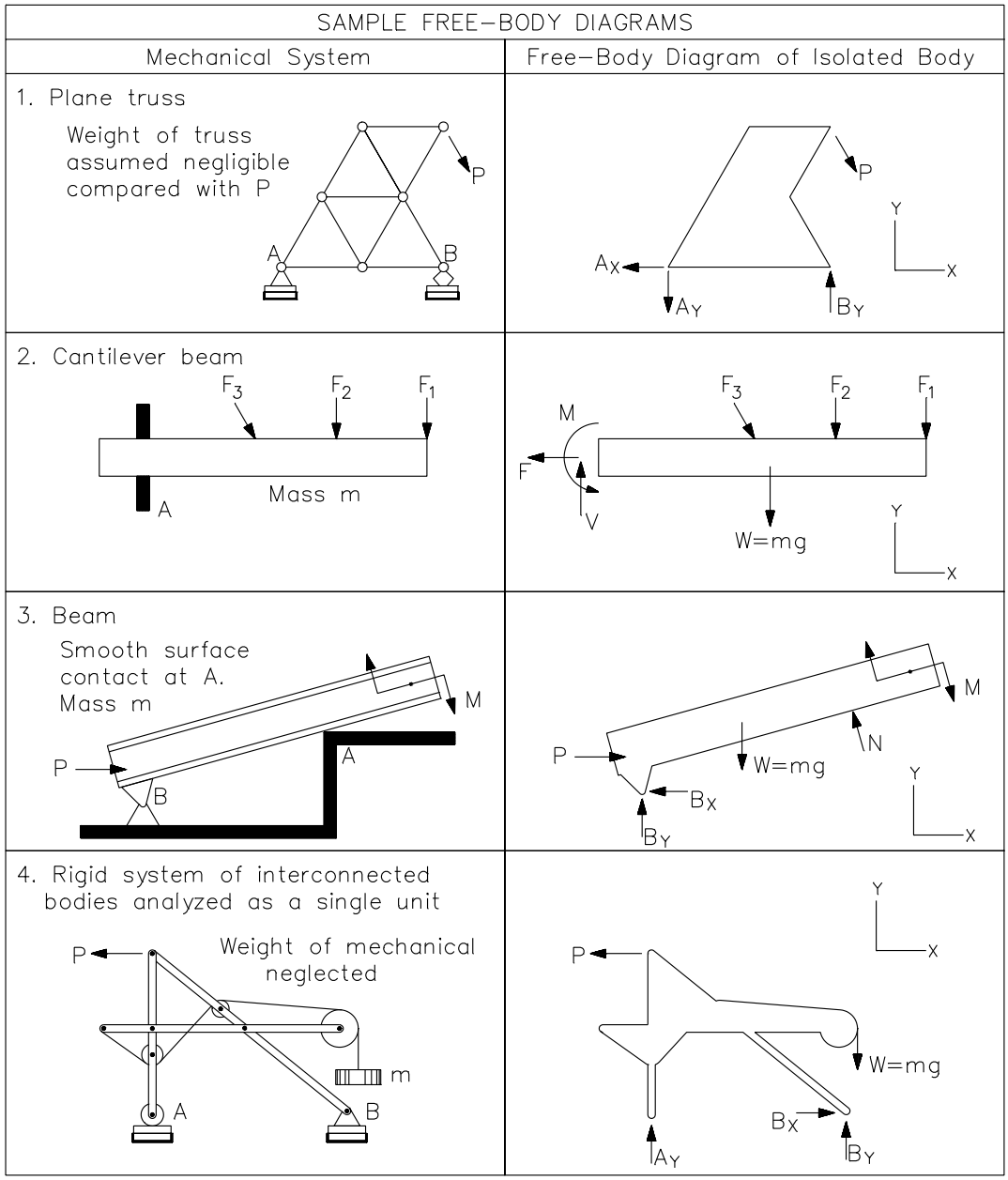

Figure 4 Various Free-Body Diagrams

Summary

The concept of the free-body diagram is summarized below.

Free-Body Diagram Summary

- A free-body diagram isolates a body and illustrates all the forces that act on the body so that a complete and accurate account of all of those forces may be considered.
- Four steps must be performed to construct a free-body diagram:
  - Determine the body or combination of bodies to be isolated.
  - Isolate the body or combination of bodies with a diagram that represents the complete external boundaries.
  - Represent all forces that act on the isolated body in their proper positions within the diagram.
  - Indicate the choice of coordinate axes directly on the diagram.

FORCE EQUILIBRIUM

Knowledge of the forces required to maintain an object in equilibrium is essential in understanding the nature of bodies at rest and in motion.

EO 1.4 STATE the conditions necessary for a body to be in force equilibrium.

EO 1.5 DEFINE the following:
a. Net force
b. Equilibrant

Net Force

When forces act on an object, the result may be a change in the object's state of motion. If certain conditions are satisfied, however, the forces may combine to maintain a state of equilibrium or balance.

To determine if a body is in equilibrium, the overall effect of all the forces acting on it must be assessed. All the forces that act on an object result in essentially one force that influences the object's motion. The force which results from all the forces acting on a body is defined as the *net force*. It is important to remember that forces are vector quantities. When analyzing various forces you must account for both the magnitude (displacement) of the force as well as the direction in which the force is applied. As described in the previous chapter, this is best done using a free-body diagram.

To understand this more clearly, consider the book resting on the table in section A of Figure 5.

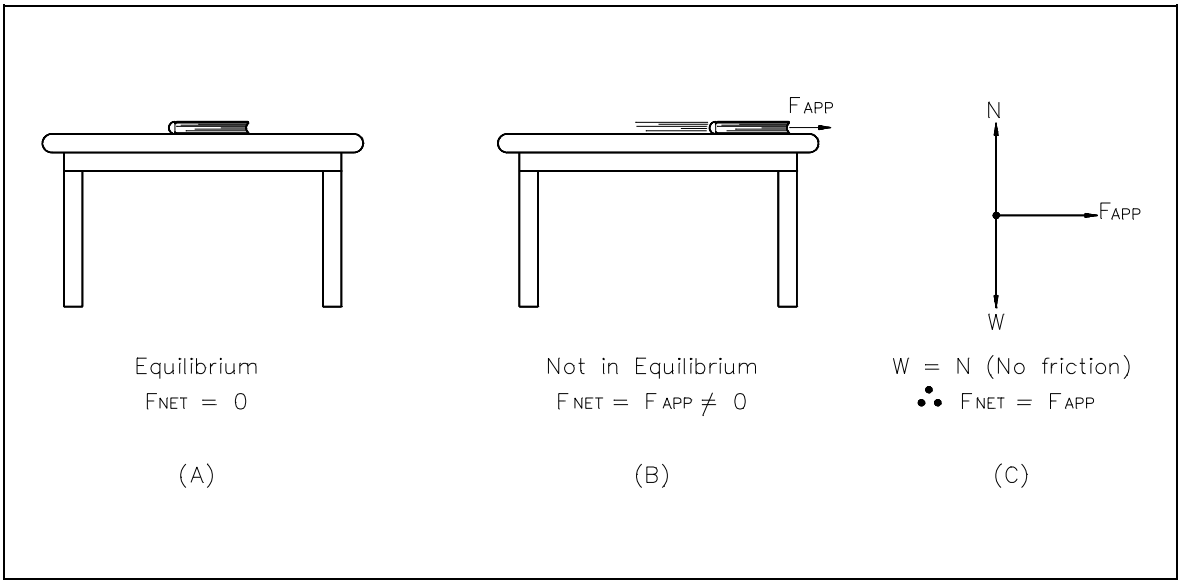

Figure 5 Net Force

The book remains stationary resting on the table because the table exerts a normal force upward equal to the weight of the book. Therefore, the net force on the book is zero. If a force is applied to the book (section B of Figure 5), and the effect of friction is neglected, the net force will be equal to the applied force, and the book will move in the direction of the applied force. The free-body diagram in section C of Figure 5 shows that the weight (W) of the book is canceled by the normal force (N) of the table since they are equal in magnitude but opposite in direction. The resultant (net) force is therefore equal to the applied force (F_{APP}).

Equilibrium

Since an object in equilibrium is considered to be in a state of balance, it can be surmised that the net force on the object is equal to zero. That is, if the vector sum of all the forces acting on an object is equal to zero, then the object is in equilibrium.

Newton's first law of motion describes equilibrium and the effect of force on a body that is in equilibrium. That law states "An object remains at rest (if originally at rest) or moves in a straight line with a constant velocity if the net force on it is zero." Newton's first law of motion is also called the law of inertia. *Inertia* is the tendency of a body to resist a change in its state of motion.

The first condition of equilibrium, a consequence of Newton's first law, may be written in vector form, "A body will be in translational equilibrium if and only if the vector sum of forces exerted on a body by the environment equals zero."

For example, if three forces act on a body it is necessary for the following to be true for the body to be in equilibrium.

$$F_1 + F_2 + F_3 = 0 \tag{4-3}$$

This equation may also be written as follows.

$$\Sigma F = 0 \tag{4-4}$$

This sum includes all forces exerted on the body by its environment. The vanishing of this vector sum is a necessary condition, called the first condition of equilibrium, that must be satisfied in order to ensure translational equilibrium. In three dimensions (*x*,*y*,*z*), the component equations of the first condition of equilibrium are:

$$\Sigma F_X = 0 \qquad \Sigma F_Y = 0 \qquad \Sigma F_Z = 0 \tag{4-5}$$

This condition applies to objects in motion with constant velocity and to bodies at rest or in static equilibrium (referred to as STATICS).

Applying the knowledge that an object in equilibrium has a net force equal to zero, the following example can be solved:

Example:

The object in Figure 6 has a weight of 125 lbf. The object is suspended by cables as shown. Calculate the tension (T_1) in the cable at 30° with the horizontal.

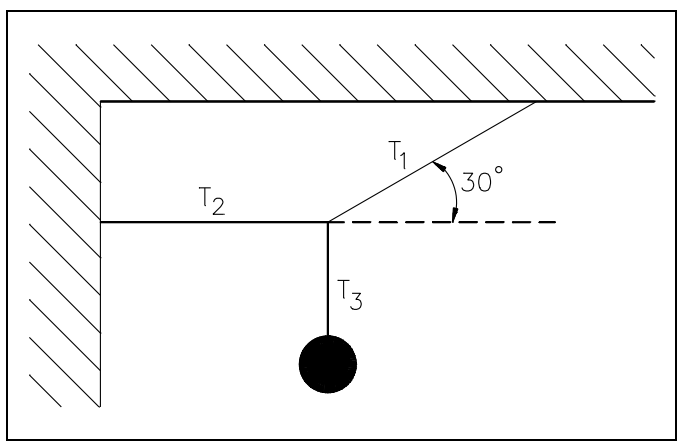

Figure 6 Hanging Object

The tension in a cable is the force transmitted by the cable. The tension at any point in the cable can be measured by cutting a suitable length from it and inserting a spring scale.

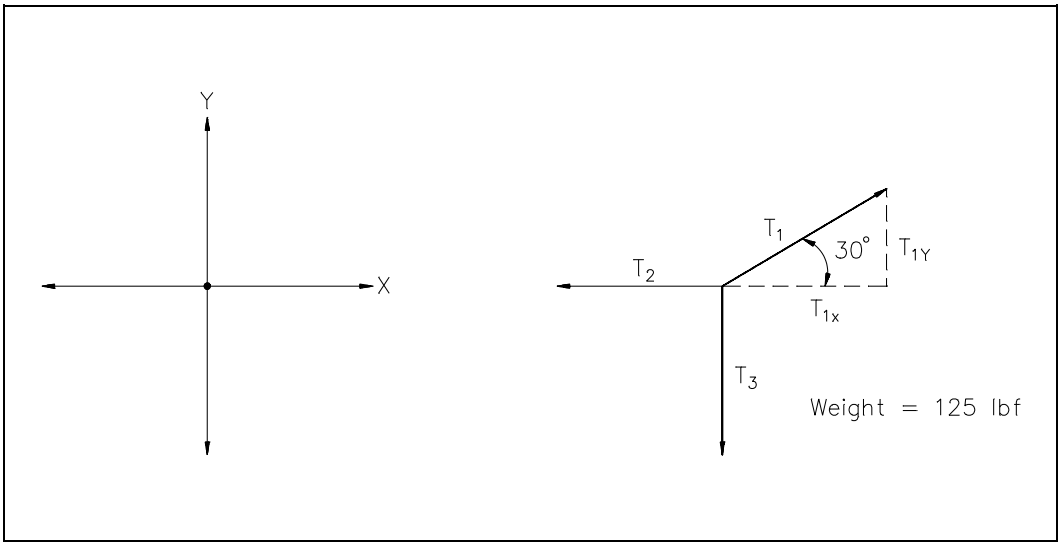

Figure 7 Free-Body Diagram

Solution:

Since the object and its supporting cables are motionless (i.e., in equilibrium), we know that the net force acting on the intersection of the cables is zero. The fact that the net force is zero tells us that the sum of the x-components of T_1, T_2, and T_3 is zero, and the sum of the y-components of T_1, T_2, and T_3 is zero.

$$\Sigma F_x = T_{1_x} + T_{2_x} + T_{3_x} = 0$$
$$\Sigma F_y = T_{1_y} + T_{2_y} + T_{3_y} = 0$$

The tension T_3 is equal to the weight of the object, 125 lbf. The *x* and *y* components of the tensions can be found using trigonometry (e.g., sine function). Substituting known values into the second equation above yields the following.

$$\begin{aligned}
\Sigma F_y = (T_1 \sin 30°) + (T_2 \sin 180°) + (T_3 \sin 270°) &= 0 \\
(T_1)(0.5) + (T_2)(0) + (125 \text{ lbf})(-1) &= 0 \\
0.5T_1 - 125 \text{ lbf} &= 0 \\
0.5T_1 &= 125 \text{ lbf} \\
T_1 &= 250 \text{ lbf}
\end{aligned}$$

A simpler method to solve this problem involves assigning a sign convention to the free-body diagram and examining the direction of the forces.

By choosing (+) as the upward direction and (-) as the downward direction, the student can determine by examination that 1) the upward component of T_1 is + T_1 sin 30º, 2) the tension T_3 is -125 lbf, and 3) T_2 has no *y*- component. Therefore, using the same equation as before, we obtain the following.

$$\sum F_y = (T_1 \sin 30°) - 125 \text{ lbf} = 0$$
$$0.5\, T_1 = 125 \text{ lbf}$$
$$T_1 = 250 \text{ lbf}$$

If the sum of all forces acting upon a body is equal to zero, that body is said to be in force equilibrium. If the sum of all the forces is not equal to zero, any force or system of forces capable of balancing the system is defined as an *equilibrant*.

Example:

A 2000 lbm car is accelerating (on a frictionless surface) at a rate of 2 ft-sec. What force must be applied to the car to act as an equilibrant for this system?

Solution: a. Draw a free-body diagram.

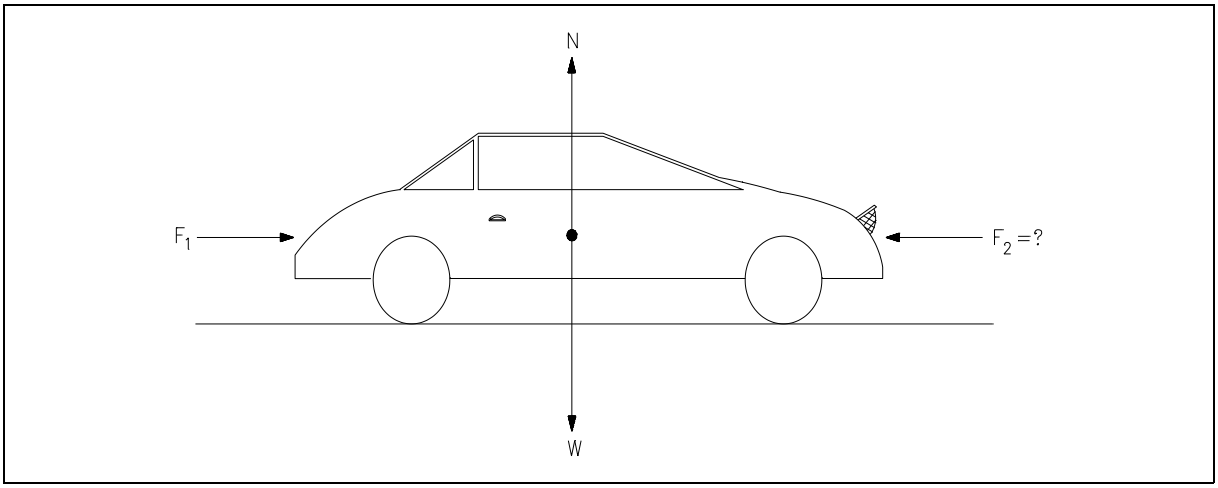

Figure 8 Free-Body Diagram

b. A Force, F_2, MUST be applied in the opposite direction to F_1 such that the sum of all forces acting on the car is zero.

$$\sum \text{Forces} = F_1 + F_2 + N + W = 0$$

c. Since the car remains on the surface, forces N and W are in equal and opposite directions. Force F_2 must be applied in an equal and opposite direction to F_1 in order for the forces to be in equilibrium.

$$F_2 = F_1 = \frac{ma}{g_c} = (2000\,\text{lbm} \times 2\,\text{ft-sec}^2) \div 32.17\ \frac{\text{ft-lbm}}{\text{lbf-sec}^2}$$
$$= 124\ \text{lbf}$$

Summary

The concepts presented in this chapter are summarized below.

Force Equilibrium Summary

- The force that is the resultant force of all forces acting on a body is defined as the net force.
- If the vector sum of all the forces acting on an object is equal to zero, then the object is in equilibrium.
- The first condition of equilibrium is stated as follows: "A body will be in translational equilibrium if and only if the vector sum of forces exerted on a body by the environment equals zero."

$$F_1 + F_2 + F_3 = 0$$
or
$$\Sigma F = 0$$

- Any force or system of forces capable of balancing a system so that the net force is zero is defined as an equilibrant.

TYPES OF FORCE

When determining how an object reacts to a force or forces, it is important to understand the different types of forces that may act on the object.

EO 1.6 **DEFINE the following:**
- **a. Tensile force**
- **b. Compressive force**
- **c. Frictional force**

EO 1.7 **EXPLAIN the difference between a static-friction force and a kinetic-friction force.**

EO 1.8 **STATE two factors that affect the magnitude of the friction force.**

EO 1.9 **EXPLAIN the difference between centripetal force and centrifugal force.**

The previous section discussed the equilibrium of forces as they act on bodies. Recalling that a force is defined as a vector quantity that tends to produce an acceleration of a body in the direction of its application, it is apparent that the student must be acquainted with the various types of forces that exist in order to construct a correct free-body diagram and apply the appropriate equation. A force is applied either by direct mechanical contact or by remote action.

Tensile and Compressive Forces

In discussing the types of forces, a simple rule is used to determine if the force is a tensile or a compressive force. If an applied force on a member tends to pull the member apart, it is said to be in *tension*. If a force tends to compress the member, it is in *compression*. It should also be mentioned that ropes, cables, etc., that are attached to bodies can only support tensile loads, and therefore such objects are in tension when placed on the free-body diagram. In addition, when a fluid is involved, it should be understood that fluid forces are almost always compressive forces.

Friction

Another type of force often used in classical physics is the force resulting from two surfaces in contact, where one of the surfaces is attempting to move parallel to or over the other surface. Such forces are referred to as *friction forces*. There are two types of friction forces: those due to dry friction, sometimes called Coulomb friction, and those resulting from fluid friction.

Fluid friction develops between layers of fluid moving at different velocities. This type of frictional force is used in considering problems involving the flow of fluids through pipes. Such problems are covered in the Fundamentals Manual on fluid flow. In this section, problems involving rigid bodies which are in contact along dry surfaces are considered.

The laws of dry friction are best understood by the following experiment. A block of weight W is placed on a horizontal plane surface (see Figure 9). The forces acting on the block are its weight W and the normal force N of the surface. Since the weight has no horizontal component, the normal force of the surface also has no horizontal component; the reaction is therefore normal to the surface and is represented by N in part (a) of the figure. Suppose now, that a horizontal force P is applied to the block (see part (b)). If P is small, the block will not move. Some other horizontal force must therefore exist which balances P. This other force is the static-friction force F, which is actually the resultant of a great number of forces acting over the entire surface of contact between the block and the plane. The nature of these forces is not known exactly, but it is generally assumed that these forces are due to the irregularities of the surfaces in contact and also to molecular action.

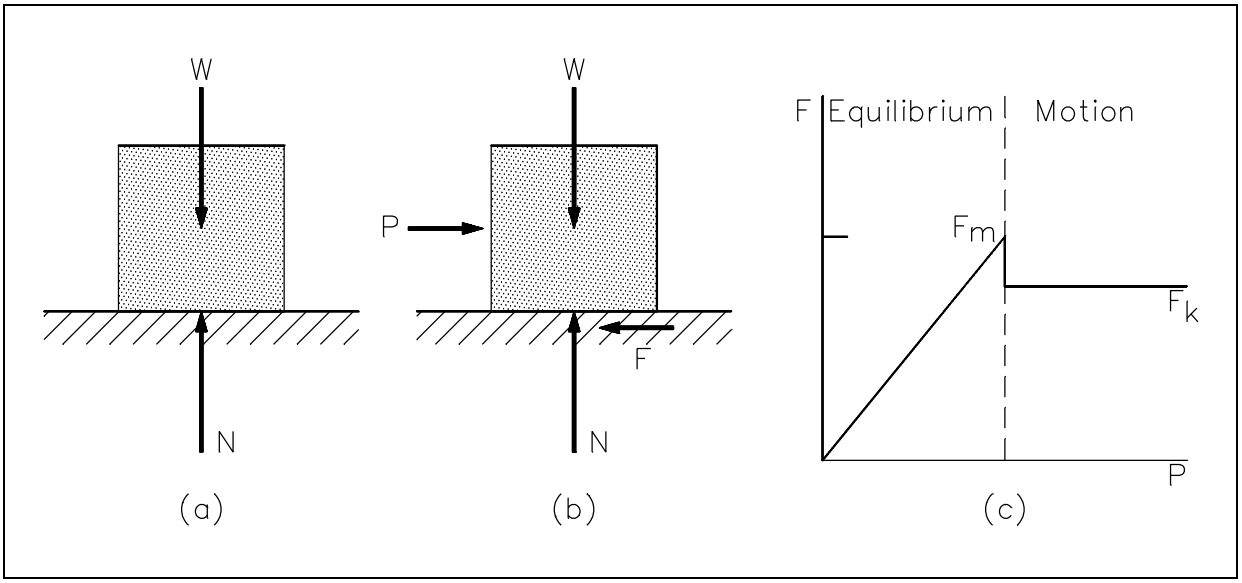

Figure 9 Frictional Forces

If the force P is increased, the friction force F also increases, continuing to oppose P, until its magnitude reaches a certain maximum value F_M (see part (c) of Figure 9). If P is further increased, the friction force cannot balance it any more, and the block starts sliding. As soon as the block has been set in motion, the magnitude of F drops from F_M to a lower value F_K. This is because there is less interpenetration between the irregularities of the surfaces in contact when these surfaces move with respect to one another. From then on, the block keeps sliding with increasing velocity (i.e., it accelerates) while the friction force, denoted by F_K and called the kinetic-friction force, remains approximately constant.

Experimental evidence shows that the maximum value F_M of the static-friction force is proportional to the normal component N of the reaction of the surface, as shown in Equation 4-5.

$$F_M = \mu_s N \tag{4-5}$$

The term μ_s is a constant called the coefficient of static friction. Similarly, the magnitude F_K of the kinetic-friction force may be expressed in the following form.

$$F_K = \mu_K N \tag{4-6}$$

The term μ_K is a constant called the coefficient of kinetic friction. The coefficients of friction, μ_S and μ_K, do not depend upon the area of the surfaces in contact. Both coefficients, however, depend strongly on the nature of the surfaces in contact. Since they also depend upon the exact condition of the surfaces, their value is seldom known with an accuracy greater than 5 percent. It should be noted that frictional forces are always opposite in direction to the motion (or impending motion) of the object.

Centripetal Force

An object moving at constant speed in a circle is not in equilibrium. Although the magnitude of the linear velocity is not changing, the direction of velocity is continually changing. Since a change in direction requires acceleration, an object moving in a circular path has a constant acceleration towards the center of the circular path.

Recalling Newton's second law of motion, F = ma, a force is required to cause acceleration. Therefore, to have constant acceleration towards the center of the circular path, there must be a net force acting towards the center. This force is known as centripetal force. Without this force, an object will move in a straight line. Figure 10 illustrates the centripetal force.

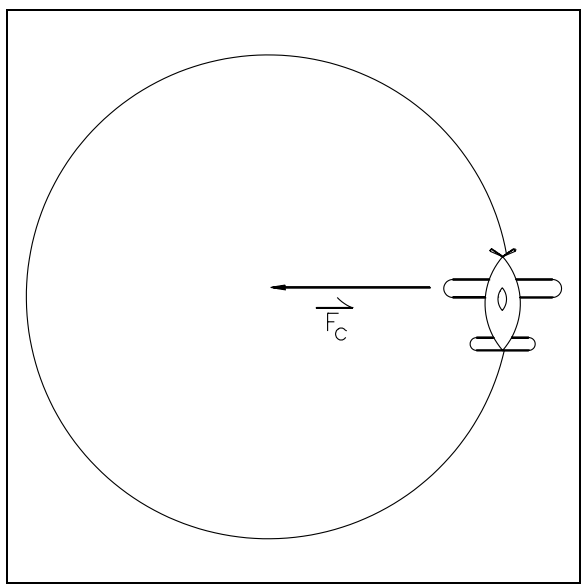

Figure 10 Centripetal Force

Centrifugal Force

Another force, which appears to be opposite the direction of motion, is the centrifugal force acting on an object that follows a curved path. This force appears to be a force directed away from the center of the circular path. This is actually a fictitious force, but is an apparent force that is used to describe the forces present due to an object's rotation.

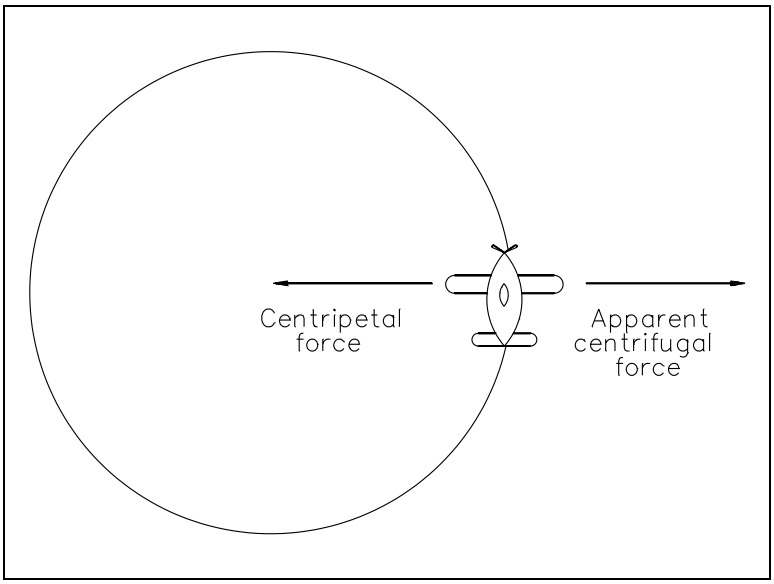

Figure 11 Centrifugal Force

To better understand centripetal and centrifugal forces, consider that a string is attached to the plane in Figure 10. As the plane rotates about the center, the string places a centripetal force on the plane. This causes the plane's velocity to change in direction, thus causing it to travel in a circle.

The apparent outward force, centrifugal force, seems to pull the plane away from the center shown in Figure 11. This is the same apparent outward force one feels when riding in a car when the car travels in a circle. It can be proven that centrifugal force is not an actual force by cutting the string. In doing so, the plane will fly off in a straight line that is tangent to the circle at the velocity it had the moment the string was cut. If there were an actual centrifugal force present, the plane would not fly away in a line tangent to the circle, but would fly directly away from the circle (see Figure 12).

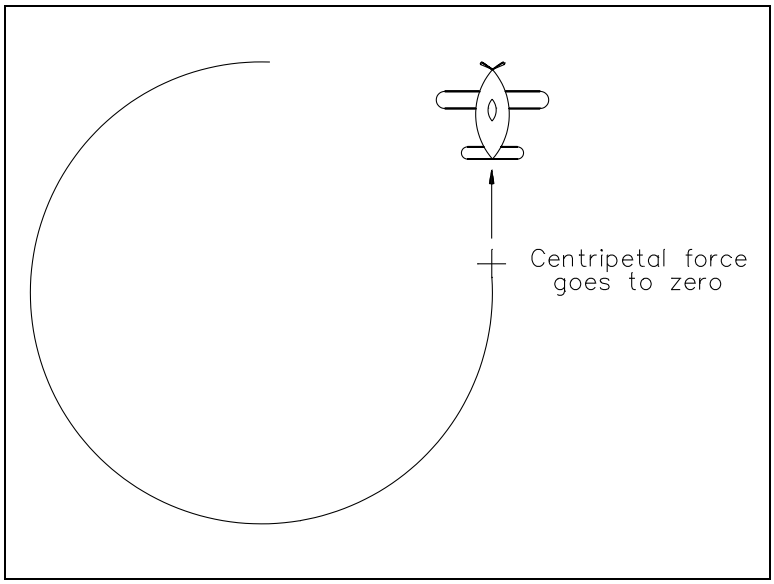

Figure 12 Loss of Centripetal Force

Summary

The concepts of this chapter are summarized below.

Types of Force Summary

- A tensile force is an applied force that tends to pull an object apart.
- A compressive force is an applied force that tends to compress an object.
- Frictional force is the force resulting from two surfaces in contact, where one of the surfaces is attempting to move with respect to the other surface.
- Static-frictional forces are those frictional forces present when an object is stationary, whereas kinetic-frictional forces are those frictional forces present between two objects that are moving.
- The magnitude of the frictional force is affected by the following:
  - Weight of the object being moved
  - Type of surface on the object being moved
  - Type of surface on which the object is moving.
- Centripetal force is the force on an object moving in a circular path that is directed towards the center of the path, whereas the centrifugal force is the fictitious force that appears to be directed away from the center of the circular path.

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Department of Energy
Fundamentals Handbook

CLASSICAL PHYSICS
Module 5
Energy, Work, and Power

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- Beiser, Arthur, Applied Physics Schaums Outline Series, McGraw-Hill Book Company, 1976.
- Murray, Raymond and Cobb, Grover, Physics Concepts and Consequences, Prentice-Hall, Inc., 1970.
- Alexander, Joseph, et al., Physics for Engineering Technology, John Wiley and Sons, 1978.
- Sears, Francis and Zemansky, Mark, University Physics, 3rd edition, Addison-Wesley Publishing Co.
- Science and Fundamental Engineering (SAFE), Classical Physics, "Measure" Combustion Engineering, Inc., 1988.
- Academic Program for Nuclear Power Plant Personnel, Volume II, General Physics Corporation, Library of Congress Card #A 397747, April 1982.

TERMINAL OBJECTIVE

1.0 Given necessary information about a system, **CALCULATE** the work performed and/or power produced or used by that system.

ENABLING OBJECTIVES

1.1 **DEFINE** the following terms:
- a. Energy
- b. Potential energy
- c. Kinetic energy
- d. Work
- e. Power

1.2 **STATE** the mathematical expression for:
- a. Potential energy
- b. Kinetic energy
- c. Work
- d. Power

1.3 For a mechanical system, **CALCULATE** energy, work, and power.

1.4 **STATE** the First Law of Thermodynamics, "Conservation of Energy."

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ENERGY AND WORK

Energy is the measure of the ability to do work or cause a change. Work is a measure of the amount of energy required to move an object.

EO 1.1 DEFINE the following terms:
- **a. Energy**
- **b. Potential energy**
- **c. Kinetic energy**
- **d. Work**

EO 1.2 STATE the mathematical expression for:
- **a. Potential energy**
- **b. Kinetic energy**
- **c. Work**

EO 1.3 For a mechanical system, CALCULATE energy, work, and power.

Energy

Energy is defined as the measure of the ability to do work. Energy determines the capacity of a system to perform work and may be stored in various forms. Some of the more basic mechanical systems involve the concepts of potential and kinetic energy. Both of these terms will be explained more fully later in this chapter. More advanced systems may include other types of energy such as chemical, electromagnetic, thermal, acoustic, and nuclear. A piledriver hammer performs work by virtue of its falling motion. Coal burned in a fossil-fueled power plant is undergoing energy release by a chemical reaction. Fuel elements in a nuclear power reactor produce energy by a nuclear reaction. For the purposes of this course, our discussions will be limited to mechanical and thermal forms of energy (e.g., heat). It should be noted, however, that the principles involved with energy calculations are similar for all types of energy.

Both thermal and mechanical energy can be separated into two categories, transient and stored. Transient energy is energy in motion, that is, energy being transferred from one place to another. Stored energy is the energy contained within a substance or object. Both of these categories of energy will be discussed in this module.

Potential Energy

Potential energy is defined as the energy stored in an object because of its position. An example is the potential energy of an object above the surface of the earth in the earth's gravitational field. Potential energy also applies to energy due to separation of electrical charge and to energy stored in a spring, in other words, energy due to position of any force field.

As an example, consider the energy stored in hydrogen and oxygen as potential energy to be released on burning. Burning changes their relative separation distance from the elemental form to the compound form as water releases the potential energy.

When discussing mechanical potential energy, we look at the position of an object. The measure of an object's position is its vertical distance above a reference point. The reference point is normally the earth's surface, but can it be any point. The potential energy of the object represents the work required to elevate the object to that position from the reference point. Potential energy is mathematically represented by Equation 5-1.

$$\text{PE} = \text{work to elevate} = \text{weight x height} = \frac{mgz}{g_c} \qquad (5\text{-}1)$$

where:

PE	=	potential energy in ft-lbf
m	=	mass in lbm
g	=	32.17 ft/sec^2
g_c	=	32.17 (lbm-ft)/(lbf-sec^2)
z	=	height above a reference in ft

It should be noted the g_c is used only when using the English system of measurement.

Example: What is the potential energy of a 50 lbm object suspended 10 feet above the ground?

$$\text{PE} = \frac{mgz}{g_c} = \left(\frac{50\ \text{lbm}}{1}\right)\left(\frac{32.17\ \text{ft}}{\text{sec}^2}\right)\left(\frac{10\ \text{ft}}{1}\right)\left(\frac{\text{lbf}-\text{sec}^2}{32.17\ \text{lbm}-\text{ft}}\right)$$

Answer: PE = 500 ft-lbf

Kinetic Energy

Kinetic energy is defined as the energy stored in an object because of its motion. If you have a baseball in your hand, it has no kinetic energy because it is not moving. But if you throw the ball, your hand has provided energy to give the ball motion. When you release the ball, it leaves your hand at some velocity. The energy you have given the ball will determine the velocity of the ball. Because the kinetic energy is due to the motion of the object, and motion is measured by velocity, kinetic energy can be calculated in terms of its velocity, as shown below.

$$\text{KE} = \frac{mv^2}{2g_c} \qquad (5\text{-}2)$$

where:

KE	=	kinetic energy in ft-lbf
m	=	mass in lbm
v	=	velocity in ft/sec
g_c	=	(32.17 lbm-ft)/(lbf-sec^2)

Example: What is the kinetic energy of a 10 lbm object that has a velocity of 8 ft/sec?

$$KE = \frac{mv^2}{2g_c} = \left(\frac{10\ \text{lbm}}{2}\right)\left(\frac{8\ \text{ft}}{\text{sec}}\right)^2\left(\frac{\text{lbf-sec}^2}{32.17\ \text{lbm-ft}}\right)$$

$$KE = (5\ \text{lbm})\left(\frac{64\ \text{ft}^2}{\text{sec}^2}\right)\left(\frac{\text{lbf-sec}^2}{32.17\ \text{lbm-ft}}\right)$$

Answer: KE = 9.95 ft-lbf

The kinetic energy of an object represents the amount of energy required to increase the velocity of the object from rest (v = 0) to its final velocity, or the work it can do as it pushes against something in slowing down (waterwheel or turbine, for example.)

Thermal Energy

Thermal energy is that energy related to temperature (the higher the temperature, the greater the molecular movement, and the greater the energy). If one object has more thermal energy than an adjacent substance, the substance at the higher temperature will transfer thermal energy (at a molecular level) to the other substance. Note that the energy is moving from one place to another (it is in motion) and is referred to as transient energy or, more commonly in the case of thermal energy, heat.

The only stored energy in a solid material is internal energy. Internal energy is the energy stored in a substance because of the motion and position of the particles of the substance. Heat and internal energy will be covered in the Fundamentals Manual on Heat Transfer, Fluid Flow, and Thermodynamics.

Mechanical Energy

Mechanical energy is energy related to motion or position. Transient mechanical energy is commonly referred to as work. Stored mechanical energy exists in one of two forms: kinetic or potential. Kinetic and potential energy can be found in both fluids and solid objects.

Work

Work is commonly thought of as any activity requiring exertion. However, the definition in physics is much more specific. *Work* is done by a force acting on a moving object if the object has some component of motion in the direction of the force. Work can be done BY a person, a machine, or an object by applying a force and causing something to move. More specifically, work is done by a force acting on a moving object if the object has some component of motion in the direction of the force. Work can be done ON an object by applying a force that causes it to move. For example, if you push on a box (apply a force) and it moves three feet, work has been performed BY you to the box, while work has been performed ON the box. If you push on the box and it does not move, then work, by our definition, has not been accomplished. Work can be defined mathematically by Equation 5-3.

$$W = F \times d \tag{5-3}$$

where:

W = work done in ft-lbf
F = force applied to the object in lbf
d = distance the object is moved (in ft.) with the force applied

Example: You push a large box for three minutes. During that time, you exert a constant force of 200 lbf to the box, but it does not move. How much work has been accomplished?

$$W = F \times d$$
$$W = 200 \text{ lbf} \times 0 \text{ ft}$$
$$W = 0 \text{ ft-lbf work done}$$

Remember that if no movement is achieved, no work has been accomplished. Even if you feel fatigued, no work has been done. Work can be thought of as what has been accomplished. If nothing is accomplished, then no work has been done.

Example: You push the same box as mentioned above. You apply a horizontal force of 200 lbf to the box, and the box moves five feet horizontally. How much work have you done?

$$W = F \times d$$
$$W = 200 \text{ lbf} \times 5 \text{ ft}$$
$$W = 1000 \text{ ft-lbf}$$

In this case, work can be described as work done by the person pushing the box or work performed on the box. In either case, the amount of work is the same.

Summary

The main points covered in this chapter are summarized below.

Energy and Work Summary

- Energy is the ability to do work.
- The work done by a force on an object is the product of the force and the distance the object moves in the direction of the force.
- Kinetic energy is the energy an object has because of its motion.
- $KE = \frac{mv^2}{2g_c}$ or $KE = \frac{1}{2} mv^2$
- Potential energy is the energy of an object due to its position.
- $PE = \frac{mgz}{g_c}$

LAW OF CONSERVATION OF ENERGY

Energy does not simply appear and disappear. Energy is transferred from one position to another or transformed from one type of energy to another.

EO 1.4 STATE the First Law of Thermodynamics, "Conservation of Energy."

Conservation of Energy

The First Law of Thermodynamics is simply stated "energy cannot be created or destroyed, only altered in form." In the previous chapter, we discussed potential energy, where a force is applied to an object, raising it from some point of origin to some height. The energy expended in raising the object is equivalent to the potential energy gained by the object because of its height. This is an example of a transfer of energy as well as an alteration of the type of energy. Another example is throwing a baseball. While the ball is in your hand, it contains no kinetic energy. You apply a force to the ball by throwing it. The ball leaves your hand with a velocity, giving it kinetic energy equal to the work applied by your hand. Mathematically, this can be described by the following simplified equation.

$$\text{Energy}_{\text{initial}} + \text{Energy}_{\text{added}} - \text{Energy}_{\text{removed}} = \text{Energy}_{\text{final}} \qquad (5\text{-}4)$$

where:

$\text{Energy}_{\text{initial}}$ is energy initially stored in an object/substance. This energy can exist in various combinations of kinetic energy and potential energy.

$\text{Energy}_{\text{added}}$ is energy added to the object/substance. Heat can be added. Energy can be added in the form of stored energy in any mass added, such as water to a fluid system. Work can be done on a system. Heat is energy gained or lost at a microscopic level. Work is the same at a macroscopic level.

$\text{Energy}_{\text{removed}}$ is energy removed from an object/substance. Heat can be rejected. Work can be done by the system. This energy can be in the form of energy stored in any mass removed.

$\text{Energy}_{\text{final}}$ is energy remaining within the object/substance after all energy transfers and transformations occur. This energy can exist in various combinations of kinetic, potential, flow, and internal energy.

To further describe each of the components of the above equation, each component can be broken down as follows:

$Energy_{initial}$ = $KE_1 + PE_1$

$Energy_{added}$ = Work done on and heat added to the system

$Energy_{removed}$ = Work done by and heat removed from the system

$Energy_{final}$ = $KE_2 + PE_2$

The resulting energy balance is provided in Equation 5-5.

$$KE_1 + PE_1 + E_{added} - E_{removed} = KE_2 + PE_2 \quad (5\text{-}5)$$

Neglecting any heat removed or added to a system, we can replace E_{added} and $E_{removed}$ in Equation 5-5 with their associated work terms to obtain Equation 5-6.

$$KE_1 + PE_1 + W_{on} = KE_2 + PE_2 + W_{by} \quad (5\text{-}6)$$

The final energy balance is called a "simplified energy balance." Any energy balance is a statement of the Law of Conservation of Energy. In this simplified form, the balance applies only to mechanical problems, since we neglected heat. However, more specific energy balances that include heat will be discussed in other Fundamental Manuals. For example, specific energy balances for flow systems will be discussed in the Heat Transfer, Fluid Flow, and Thermodynamics modules.

Summary

The law of conservation of energy is summarized below.

Conservation of Energy Summary

- Energy cannot be created or destroyed, only altered in form.
- Simplified energy balance:

  $KE_1 + PE_1 + E_{added} = KE_2 + PE_2 + E_{removed}$

POWER

Power is a measure of the rate at which energy is used. Thermal power is the term used to refer to the transfer of heat. Mechanical power is the term used to describe when work is being done.

EO 1.1 DEFINE the following terms:
e. Power

EO 1.2 STATE the mathematical expression for:
d. Power

EO 1.3 For a mechanical system, CALCULATE energy, work, and power.

Power

Power is defined as the amount of energy used per unit time or rate of doing work. It has the units of watt, Btu, horsepower, or ft-lbf/sec.

Thermal Power

Thermal power is the measure of thermal energy used per unit time. It is the rate of heat transfer or heat flow rate. Examples of thermal power units are British Thermal Units (Btu) or kilowatts (Kw). Thermal power is calculated basically by the mathematical expression of:

$$\text{Thermal Power} = \frac{\text{heat used}}{\text{time required}}$$

Thermal energy and calculations of thermal power will be covered in more detail in the Fundamentals Manual for Heat Transfer, Fluid Flow, and Thermodynamics.

Mechanical Power

Mechanical energy used per unit time is called *mechanical power*. Mechanical power is the rate at which work is done. Mechanical power is expressed in units of joules/sec (joules/s) or a watt (W) in the mks system, and feet - pounds force per second (ft-lbf/s) or horsepower (hp) in the English system. Mechanical power can be calculated using the following mathematical expression.

$$\text{Power} = \frac{\text{work done}}{\text{time required}}$$

Because work can be defined as force times distance, we can also use the following equation:

$$P = \frac{Fd}{t} \tag{5-6}$$

where:

P = Power (W or ft-lbf/s)
F = Force (N or lbf)
d = distance (m or ft)
t = time (sec)

One horsepower is equivalent to 550 ft-lbf/s and 745.7 watts. Because in the equation above *d* divided by *t* is the same as velocity, an alternate description of power is as follows.

$$P = \frac{Fv}{550} \tag{5-7}$$

where:

P = power (hp)
F = force (lbf)
v = velocity (ft/s)

When using equations 5-6 or 5-7, you must either assume force and velocity are constant or that average values of the force and velocity are used.

Example 1: A pump provides a flow rate of 10,000 gpm. The pump does 1.5 x 10^8 ft-lbf of work every 100 minutes. What is the power of the pump in hp?

$$\text{Power} = \frac{\text{work done}}{\text{time required}}$$

$$P = \left(\frac{1.5 \text{ x } 10^8 \text{ ft-lbf}}{100 \text{ min}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right)\left(\frac{1 \text{ hp}}{550 \text{ ft-lbf/sec}}\right)$$

$$P = 45.5 \text{ hp}$$

Example 2: A boy rolls a ball with a steady force of 1 lbf, giving the ball a constant velocity of 5 ft/s. What is the power used by the boy in rolling the ball?

$$P = \frac{Fv}{550}$$

$$P = \frac{(1 \text{ lbf})(5 \text{ ft/sec})}{550}$$

$$P = 9 \text{ x } 10^{-3} \text{ hp}$$

Example 3: A race car traveling at constant velocity can go one quarter mile (1455 ft) in 5 seconds. If the motor is generating a force of 1890 lbf pushing the car, what is the power of the motor in hp? Assume the car is already at full speed at t=0.

$$P = \frac{Fd}{t}$$

$$P = \left[\frac{(1890\ \text{lbf})(1455\ \text{ft})}{5\ \text{sec}}\right]\left[\frac{1\ \text{hp}}{550\ \text{ft-lbf/sec}}\right]$$

$$P = 1000\ \text{hp}$$

or

$$P = \frac{Fv}{550}$$

$$P = \frac{(1890\ \text{lbf})(291\ \text{ft/sec})}{550}$$

$$P = 1000\ \text{hp}$$

Summary

The main points of this chapter are summarized below.

Power Summary

- Power is the amount of energy used per unit time.
- $\text{Power} = \dfrac{\text{work done}}{\text{time required}}$