MATHEMATICS
Module 1
Review of Introductory Mathematics

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TERMINAL OBJECTIVE

1.0 Given a basic mathematical problem, **SOLVE** for the answer with or without the aid of a calculator.

ENABLING OBJECTIVES

1.1 **IDENTIFY** the following basic mathematical symbols and definitions.

a.	$=$	equals
b.	$\neq$	is not equal to
c.	$\equiv$	is defined as
d.	$\pm$	plus or minus
e.	$\sqrt[n]{a}$	nth root of a
f.	$\lvert a \rvert$	absolute value of a
g.	$\sum_{i=1}^{N} x_i$	sum of N values
h.	$\angle$	angle
i.	%	percent
j.	x, $\cdot$, *	multiplied by
k.	÷, /	divided by
l.	$\geq$	greater than or equal to
m.	$\leq$	less than or equal to
n.	><, <>	is not equal to (computer)
o.	∞	infinity
p.	$\propto$	is proportional to
q.	$\approx$	approximately equal to
r.	$\perp$	perpendicular to
s.	$\parallel$	parallel to

1.2 **APPLY** one of the arithmetic operations of addition, subtraction, multiplication, and division using whole numbers.

1.3 Given a set of numbers, **CALCULATE** the average value.

1.4 **APPLY** one of the arithmetic operations of addition, subtraction, multiplication, and division using fractions.

1.5 **APPLY** one of the arithmetic operations of addition, subtraction, multiplication, and division of fractions by conversion to decimal form using a calculator.

ENABLING OBJECTIVES (Cont.)

1.6 **APPLY** one of the arithmetic operations of addition, subtraction, multiplication, and division using decimals.

1.7 **APPLY** one of the arithmetic operations of addition, subtraction, multiplication, and division using signed numbers.

1.8 **DETERMINE** the number of significant digits in a given number.

1.9 Given a formula, **CALCULATE** the answer with the appropriate number of significant digits.

1.10 **CONVERT** between percents, decimals, and fractions.

1.11 **CALCULATE** the percent differential.

1.12 **APPLY** one of the arithmetic operations of addition, subtraction, multiplication, and division using exponential numbers.

1.13 Given the data, **CONVERT** integers into scientific notation and scientific notation into integers.

1.14 **APPLY** one of the arithmetic operations of addition, subtraction, multiplication, and division to numbers using scientific notation.

1.15 **CALCULATE** the numerical value of numbers in radical form.

TERMINOLOGY

This chapter reviews the terminology and associated symbols used in mathematics.

EO 1.1 IDENTIFY the following basic mathematical symbols and definitions.

a.	=	**equals**	**k.**	÷, /	**divided by**
b.	≠	**is not equal to**	**l.**	$\geq$	**greater than or equal to**
c.	≡	**is defined as**	**m.**	$\leq$	**less than or equal to**
d.	±	**plus or minus**	**n.**	><, <>	**is not equal to (computer)**
e.	$\sqrt[n]{a}$	**nth root of a**			
f.	$\lvert a \rvert$	**absolute value of a**	**o.**	∞	**infinity**
g.	$\sum_{i=1}^{N} x_i$	**sum of N values**	**p.**	∝	**is proportional to**
h.	∠	**angle**	**q.**	≈	**approximately equal to**
i.	%	**percent**	**r.**	⊥	**perpendicular to**
j.	x, · , *	**multiplied by**	**s.**	∥	**parallel to**

In order to understand and communicate in mathematical terms and to lay the foundation for the concepts and principles presented in this material, certain terms and expressions must be defined. This section covers basic definitions used in mathematics. Once understood, such knowledge should provide the foundation from which the principles of mathematics can be presented. By no means are the terms here all inclusive; they are representative of those found within the nuclear field.

<u>Equals</u>
An expression indicating values which are identical in mathematical value or logical denotation. It is given the symbol =.

<u>Is Not Equal to</u>
An expression indicating values which are not identical in mathematical value or logical denotation. It is given the symbol ≠ or ><, >< (computer).

Is defined as

A mathematical expression for defining a symbol or variable in mathematics. It is usually given the symbol ≡ .

Plus or Minus

While plus (+) and minus (-) are used individually to indicate addition and subtraction, this form is used to denote a control band, or tolerance band, or error band, such as 100 $\pm$ 5 psig. It is given the symbol $\pm$.

n^{th} root

For any integer (n greater than one), the n^{th} root ($\sqrt[n]{a}$) of a is defined as follows: $\sqrt[n]{a} = b$ if, and only if, $b^n = a$. The number n, in $\sqrt[n]{a}$, is called the index of the root. The nth root of a number (a) is a number (b) which has the property that the product of n values of b is a. For example, the third (or cube) root of 8 is 2, because 2x2x2 equals 8.

Absolute Value of a

This expression represents the magnitude of a variable without regard to its sign. It signifies the distance from zero on a number line. That is, the absolute value of -6 is 6 because -6 is 6 units from zero. Likewise, the absolute value of +6 is 6 because it, too, is 6 units from zero. It is given the symbol $|A|$ where A is any number or variable.

Sum of N values

$\sum_{i=1}^{N} x_i$ indicates the sum of numbered (indexed) values. For example, if the x_i are grades for the individual students in a class, the sum of the x_i (grades) for the students in the class of N students, divided by N, gives the average grade.

Angle

An angle is a set of points consisting of two rays with a common midpoint. It is given the symbol $\angle$.

Percent

An expression used to indicate a fraction of the whole, such as 50% of 90 is 45. It is given the symbol %.

Multiplied by

A mathematical operation that, at its simplest, is an abbreviated process of adding an integer to itself a specified number of times. It is given the symbols x, · , or * (computer).

Divided by
A mathematical process that subjects a number to the operation of finding out how many times it contains another number. It is given the symbol ÷ or /.

Greater than or equal to
It is given the symbol $\geq$, and denotes one quantity is equal to or larger than another.

Less than or equal to
It is given the symbol $\leq$, and denotes one quantity is equal to or smaller than another.

Infinity
A mathematical expression meaning very large in magnitude or distance. It is so large that it cannot be measured. It is given the symbol ∞ .

Is Proportional to
The statement that a is proportionl to b ($a \propto b$) means that a = (some constant) x b. For example, the dollars you earn in a week (straight rate) are proportional to the hours you work, with the constant being the dollars per hour you earn.

Approximately Equal to
An expression indicating a value which is not exact, but rather close to the value. It is given the symbol $\approx$.

Perpendicular to
This expression means that two objects are at right angles (form a 90-degree angle) to each other. It is given the symbol $\perp$.

Parallel to
Two lines extending in the same direction which are everywhere equidistant and not meeting. It is given the symbol $\parallel$.

Summary

The important information from this chapter is summarized below.

Terminology Summary

- This chapter reviewed the terminology needed in the application and study of mathematics.

CALCULATOR OPERATIONS

This chapter gives the student a chance to reacquaint himself with basic calculator operations.

The teaching of the "mechanics of mathematics" (division, multiplication, logarithms, etc.) in recent years has focused more on the skills of using a calculator than on the pure principles of the basic subject material. With the decreased cost of hand calculators, virtually every person owns, or has access to, a calculator. A nuclear plant operator would be wise to learn how to use most of the calculators available today. Such knowledge will help the operator make quick decisions when circumstances arise for the need of a "quick calculation" of flow rate or some other parameter.

Many calculators are available on the market today, and each one is a little different. For the purpose of this module, a scientific calculator will be needed. The Texas Instruments scientific calculator TI-30 will be used for the examples in this module. Most calculators work on the same principles, but some do not. Some calculators operate on a programming principle like Hewlett-Packard (HP). An HP calculator does not use an equal key. To perform a mathematical operation, the first number is inserted, the ENTER key is pressed, the second number is inserted, and then the mathematical function key is pressed. The result will be displayed. If a different calculator is used, the student will need to refer to the reference manual for his or her calculator.

The following section will review the general use function keys on a TI-30 calculator. In each following chapter of this module, the applicable calculator operations will be addressed.

Appendix A of this module gives a representation of a TI-30 keyboard to assist the student.

Keys

Clear entry/Clear key

ON/C — Pressing this key once will clear the last operation and the display. Pressing this key twice will clear all operations except the memory.

Note: To clear the memory, press clear then STO.

Note: Many brands break this function into two separate keys, usually labeled "clear" and "all clear," where the "clear" key clears the last entry and the "all clear" key clears the display and all pending operations.

Memory Key

STO

The TI-30 has only one memory. Pressing the STO key enters the displayed number into memory. Any number already in memory will be overwritten.

Note: Calculators with more than one memory will require a number to be entered with the STO key. For example, STO 01 means store the displayed number in memory 01; STO 20 means store the number in memory 20.

Memory Recall Key

RCL

Pressing the RCL key will retrieve the number in memory and display it. Note that the number is also still in memory. This allows the number to be used again. Pressing the RCL will also overwrite any number previously displayed.

Note: Calculators with more than one memory will require a number to be entered with the RCL key. RCL 01 means recall the number stored in the 01 memory. RCL 20 means recall the number stored in memory 20.

Constant Key

K

Certain calculations often contain repetitive operations and numbers. The K, constant, is a time-saving function that allows a single key stroke to perform a single operation and number on the displayed number.

For example, if 20 numbers are to be multiplied by -17.35, the K key can be used. Enter -17.35, then press the times key, then the K key; this "teaches" the calculator the required operation. From this point on when entering a number and pressing the K key, the calculator will automatically multiply the displayed number by -17.35, saving you six key strokes.

Summation Key

SUM

If a long list of numbers is to be added, the summation key will save time if used. Pressing the summation key adds the displayed number to the number in memory. The final sum is then retrieved from memory.

Memory Exchange Key

EXC/$x!$

The EXC, memory exchange key, swaps the displayed number with the number in memory.

Reciprocal Key

$1/x$

When pressed, it divides the displayed number into one.

FOUR BASIC ARITHMETIC OPERATIONS

This chapter reviews the basic mathematical operations of addition, subtraction, multiplication, and division of whole numbers.

EO 1.2 APPLY one of the arithmetic operations of addition, subtraction, multiplication, and division using whole numbers.

Calculator Usage, Special Keys

This chapter requires the use of the +, -, x, ÷, and = keys. When using a TI-30 calculator, the number and operation keys are entered as they are written. For example, the addition of 3 plus 4 is entered as follows:

3 key, + key, 4 key, = key, the answer, 7, is displayed

Parentheses

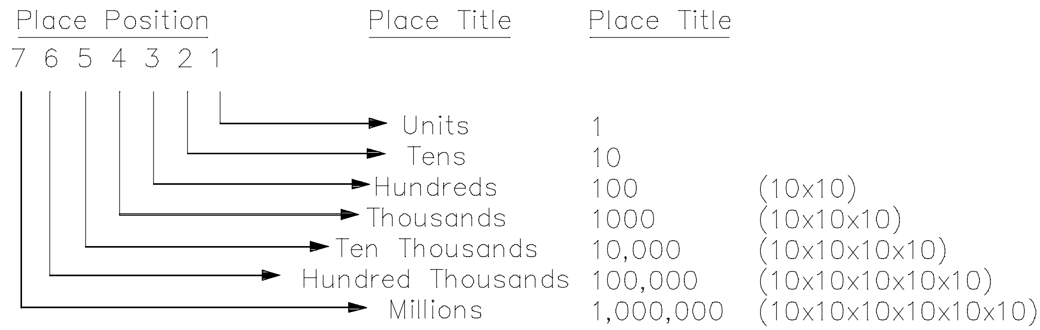
(

The parentheses keys allow a complicated equation to be entered as written. This saves the time and effort of rewriting the equation so that multiplication/division is performed first and addition/subtraction is performed second, allowing the problem to be worked from left to right in one pass.

)

The Decimal Numbering System

The decimal numbering system uses ten symbols called digits, each digit representing a number. These symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. The symbols are known as the numbers zero, one, two, three, etc. By using combinations of 10 symbols, an infinite amount of numbers can be created. For example, we can group 5 and 7 together for the number 57 or 2 and 3 together for the number 23. The place values of the digits are multiples of ten and given place titles as follows:



Numbers in the decimal system may be classified as integers or fractions. An integer is a whole number such as 1, 2, 3, . . . 10, 11, . . . A fraction is a part of a whole number, and it is expressed as a ratio of integers, such as 1/2, 1/4, or 2/3.

An even integer is an integer which can be exactly divided by 2, such as 4, 16, and 30. All other integers are called odd, such as 3, 7, and 15. A number can be determined to be either odd or even by noting the digit in the units place position. If this digit is even, then the number is even; if it is odd, then the number is odd. Numbers which end in 0, 2, 4, 6, 8 are even, and numbers ending in 1, 3, 5, 7, 9 are odd. Zero (0) is even.

Examples:

Determine whether the following numbers are odd or even: 364, 1068, and 257.

Solution:

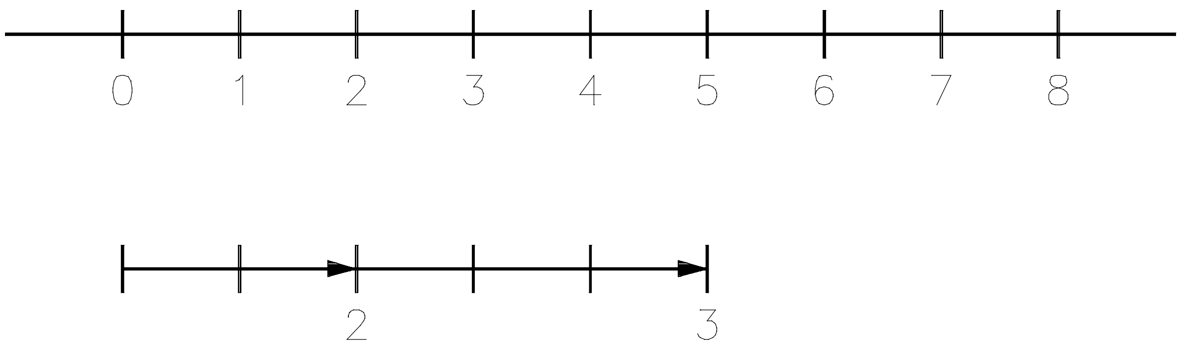
1. 364 is even because the right-most digit, 4, is an even number.
2. 1068 is even because the right-most digit, 8, is an even number.
3. 257 is odd because the right-most digit, 7, is an odd number.

Adding Whole Numbers

When numbers are added, the result is called the sum. The numbers added are called addends. Addition is indicated by the plus sign (+). To further explain the concept of addition, we will use a number line to graphically represent the addition of two numbers.

Example: Add the whole numbers 2 and 3.

Solution: Using a line divided into equal segments we can graphically show this addition.



Starting at zero, we first move two places to the right on the number line to represent the number 2. We then move an additional 3 places to the right to represent the addition of the number 3. The result corresponds to the position 5 on the number line. Using this very basic approach we can see that 2 + 3 = 5. Two rules govern the addition of whole numbers.

The commutative law for addition states that two numbers may be added in either order and the result is the same sum. In equation form we have:

$$a + b = b + a \tag{1-1}$$

For example, 5 + 3 = 8 OR 3 + 5 = 8. Numbers can be added in any order and achieve the same sum.

The associative law for addition states that addends may be associated or combined in any order and will result in the same sum. In equation form we have:

$$(a + b) + c = a + (b + c) \tag{1-2}$$

For example, the numbers 3, 5, and 7 can be grouped in any order and added to achieve the same sum:

(3 + 5) + 7 = 15 OR 3 + (5 + 7) = 15

The sum of both operations is 15, but it is not reached the same way. The first equation, (3 + 5) + 7 = 15, is actually done in the order (3 + 5) = 8. The 8 is replaced in the formula, which is now 8 + 7 = 15.

The second equation is done in the order (5 + 7) = 12, then 3 + 12 = 15. Addition can be done in any order, and the sum will be the same.

When several numbers are added together, it is easier to arrange the numbers in columns with the place positions lined up above each other. First, the units column is added. After the units column is added, the number of tens is carried over and added to the numbers in the tens column. Any hundreds number is then added to the hundreds column and so on.

Example:

Add 345, 25, 1458, and 6.

Solution:

$$\begin{array}{r} 345 \\ 25 \\ 1458 \\ \underline{+\ \ 6} \\ 1834 \end{array}$$

When adding the units column, 5 + 5 + 8 + 6 = 24. A 4 is placed under the units column, and a 2 is added to the tens column.

Then, 2 + 4 + 2 + 5 = 13. A 3 is placed under the tens column and a 1 is carried over to the hundreds column. The hundreds column is added as follows: 1 + 3 + 4 = 8.

An 8 is placed under the hundreds column with nothing to carry over to the thousands column, so the thousands column is 1. The 1 is placed under the thousands column, and the sum is 1834. To verify the sum, the numbers should be added in reverse order. In the above example, the numbers should be added from the bottom to the top.

Subtracting Whole Numbers

When numbers are subtracted, the result is called the remainder or difference. The number subtracted is called the subtrahend; the number from which the subtrahend is subtracted is called the minuend. Subtraction is indicated by the minus sign (-).

$$\begin{array}{rl} 86 & \text{Minuend} \\ \underline{-34} & -\underline{\text{Subtrahend}} \\ 52 & \text{Remainder or Difference} \end{array}$$

Unlike addition, the subtraction process is neither associative nor commutative. The commutative law for addition permitted reversing the order of the addends without changing the sum. <u>In subtraction, the subtrahend and minuend cannot be reversed</u>.

$$a - b \neq b - a \qquad (1\text{-}3)$$

Thus, the difference of 5 - 3 is not the same as 3 - 5. The associative law for addition permitted combining addends in any order. In subtraction, this is not allowed.

$$(a\text{-}b)\text{-}c \neq a\text{-}(b\text{-}c)$$

Example: $(10\text{-}5)\text{-}1 \neq 10\text{-}(5\text{-}1)$

$$4 \neq 6$$

When subtracting two numbers, the subtrahend is placed under the minuend with the digits arranged in columns placing the units place under the units place, the tens column next, and so on.

Example:

Subtract 32 from 54.

Solution:

$$\begin{array}{r} 54 \\ \underline{-32} \\ 22 \end{array}$$

Whenever the digit in the subtrahend is larger than the digit in the minuend in the same column, one place value is borrowed from the next digit to the left in the minuend. Refer to the following example.

Example:

Subtract 78 from 136.

Solution:

$$\begin{array}{r} 1\overset{2}{\cancel{3}}6 \\ \underline{-\ 78} \\ 58 \end{array}$$

When subtracting the units column, 6 - 8, a 10 is borrowed from the tens column. This now makes subtracting the units column 16 - 8. An 8 is placed under the units column. Next the tens column is subtracted.

A 10 was borrowed from the tens column and now 7 is subtracted from 12, not 13. This yields: 12 - 7 = 5. The 5 is placed under the tens column and the difference is 58.

This can be done for any subtraction formula. When the digit in the subtrahend column is larger than the digit in the minuend in the same column, a number from the next higher place position column is "borrowed." This reduces the higher position column by one.

Subtraction can be verified by adding the difference to the subtrahend, which should result in the minuend.

Multiplying Whole Numbers

Multiplication is the process of counting a number two or more times. It can be considered a shortened form of addition. Thus, to add the number 4 three times, 4 + 4 + 4, we can use multiplication terms, that is, 4 multiplied by 3. When numbers are multiplied, the result is called the product. The numbers multiplied are called factors. One factor is called the multiplicand; the other is called the multiplier. Multiplication is indicated by the times or multiplication sign (x), by a raised dot (·), or by an asterick (*).

9	Multiplicand
x 4	x Multiplier
36	Product

In multiplying several numbers, the same product is obtained even if the numbers are multiplied in a different order or even if some of the numbers are multiplied together before the final multiplication is made. These properties are called the commutative and associative laws for multiplication.

The commutative law for multiplication states that numbers can be multiplied in any order, and the result is the same product. In equation form:

$$a \times b = b \times a \tag{1-4}$$

Thus, the product of 8 x 3 is the same as 3 x 8.

The associative law for multiplication states that factors can be associated in any order, and the result is the same product. In equation form:

$$a \times (b \times c) = (a \times b) \times c \tag{1-5}$$

Thus, the numbers 2, 3, and 5 can be multiplied by first multiplying 2 x 3 to equal 6 and then multiplying 6 x 5 to equal 30. The equation may also be solved by first multiplying 3 x 5 to equal 15, and then multiplying 15 x 2 to equal 30. In either case, the product is 30.

In multiplying two numbers, one number is placed under the other with the digits arranged in columns placing units under the units place, tens under the tens place, and so on. Usually, the larger number is considered the multiplicand and the smaller number is considered the multiplier. The digit in the units place of the multiplier is multiplied first, the digit in the tens place of the multiplier next, and so on.

Example 1:

Multiply 432 by 8.

Solution:

$$\begin{array}{r} 432 \\ \times \quad 8 \\ \hline 3{,}456 \end{array}$$

In multiplying the multiplier in the units column to the multiplicand, 8 x 2 = 16. A 6 is placed under the units column, and 1 ten is carried. Then, 8 x 3 = 24, plus the 1 carried over equals 25.

A 5 is placed under the tens column, and 2 hundreds are carried over. Next, 8 x 4 = 32, plus 2 carried over, equals 34. A 4 is placed under the hundreds column and a 3 under the thousands column.

Example 2:

What is the product of 176 x 59?

Solution:

$$\begin{array}{rl} 176 & \\ \text{x } 59 & \\ \hline 1584 & \text{Multiplication by 9} \\ 880 & \text{Multiplication by 50} \\ \hline 10384 & \end{array}$$

Start by multiplying the digit in the units place of the multiplier, 9 x 6 = 54. A 4 is placed under the units column, and 5 tens are carried over.

Next, 9 x 7 = 63, plus the 5 carried over, equals 68. An 8 is placed under the tens column, and 6 hundreds are carried over. Then, 9 x 1 = 9, plus 6 carried over, equals 15. A 5 is placed under the hundreds column and a 1 under the thousands column.

The digit in the tens place of the multiplier is multiplied now: 5 x 6 = 30. Since the 5 in 59 is in the tens column, the zero is placed under the tens column, and 3 tens are carried over. Next, 5 x 7 = 35, plus the 3 carried over, equals 38. An 8 is placed under the hundreds column, and 3 hundreds are carried over.

Then, 5 x 1 = 5, plus 3 carried over, equals 8. An 8 is placed under the thousands column. The results of 176 multiplied by 9 and 50 are then added to give the final product.

Dividing Whole Numbers

Division is the process of determining how many times one number is contained in another number. When numbers are divided, the result is the quotient and a remainder. The remainder is what remains after division. The number divided by another number is called the dividend; the number divided into the dividend is called the divisor. Division is indicated by any of the following:

a division sign (÷)

a division sign ($\overline{)\quad}$)

a horizontal line with the dividend above the line and the divisor below the line $\left(\frac{\#}{\#}\right)$

a slanting line a/b meaning a divided by b

Thus, the relationship between the dividend, divisor, and quotient is as shown below:

37	Dividend
÷ 4	Divisor
9	Quotient
1	Remainder

$$\begin{array}{l} \text{Quotient} \rightarrow 9 \quad 1 \leftarrow \text{Remainder} \\ \qquad\qquad 4\overline{)\,37} \\ \text{Divisor}\uparrow \quad \uparrow\text{Dividend} \end{array}$$

Unlike multiplication, the division process is neither associative nor commutative. The commutative law for multiplication permitted reversing the order of the factors without changing the product. <u>In division the dividend and divisor cannot be reversed</u>.

Using the equation form:

$$a \div b \neq b \div a \tag{1-6}$$

For example, the quotient of 18 ÷ 6 is not the same as the quotient of 6 ÷ 18. 18 divided by 6 equals 3; 6 divided by 18 equals 0.33.

The associative law for multiplication permitted multiplication of factors in any order. In division, this is not allowed.

$$(a \div b) \div c \neq a \div (b \div c)$$

Example: $(8 \div 4) \div 2 \neq 8 \div (4 \div 2)$

$$1 \neq 4$$

When dividing two numbers, the divisor and dividend are lined up horizontally with the divisor to the left of the dividend. Division starts from the left of the dividend and the quotient is written on a line above the dividend.

Example 1:

Divide 347 by 5.

Solution:

$$\begin{array}{rl} 69 & \\ 5\overline{)\,347} & \\ \underline{30\;\,} & \\ 47 & \\ \underline{45} & \\ 2 & \text{Remainder} \end{array}$$

Starting from the left of the dividend, the divisor is divided into the first digit or set of digits it divides into. In this case, 5 is divided into 34; the result is 6, which is placed above the 4.

This result (6) is then multiplied by the divisor, and the product is subtracted from the set of digits in the dividend first selected. 6 x 5 equals 30; 30 subtracted from 34 equals 4.

The next digit to the right in the dividend is then brought down, and the divisor is divided into this number. In this case, the 7 is brought down, and 5 is divided into 47; the result is 9, which is placed above the 7.

Again, this result is multiplied by the divisor, and the product is subtracted from the last number used for division. 9 x 5 equals 45; 45 subtracted from 47 equals 2. This process is repeated until all of the digits in the dividend have been brought down. In this case, there are no more digits in the dividend. The result of the last subtraction is the remainder. The number placed above the dividend is the quotient. In this case, 347 ÷ 5 yields a quotient of 69 with a remainder of 2.

Example 2:

Divide 738 by 83.

Solution:

$$
\begin{array}{rrl}
 & 8 & \\
83 \big) & \overline{738} & \\
 & \underline{664} & \\
 & 74 & \text{Remainder}
\end{array}
$$

Example 3:

Divide 6409 by 28.

Solution:

$$
\begin{array}{rrl}
 & 228 & \\
28 \big) & \overline{6409} & \\
 & \underline{56} & \\
 & 80 & \\
 & \underline{56} & \\
 & 249 & \\
 & \underline{224} & \\
 & 25 & \text{Remainder}
\end{array}
$$

Division can be verified by multiplying the quotient by the divisor and adding the remainder. The result should be the dividend. Using Example 3, multiply 228 by 28 to check the quotient.

$$
\begin{array}{rl}
228 & \\
\underline{\times\ 28} & \\
1824 & \\
\underline{456} & \\
6384 & \rightarrow \text{ Product} \\
\underline{+\ \ 25} & \rightarrow \text{ Remainder of 25} \\
6409 &
\end{array}
$$

Hierarchy of Mathematical Operations

Mathematical operations such as addition, subtraction, multiplication, and division are usually performed in a certain order or sequence. Typically, multiplication and division operations are done prior to addition and subtraction operations. In addition, mathematical operations are also generally performed from left to right using this heirarchy. The use of parentheses is also common to set apart operations that should be performed in a particular sequence.

Example:

Perform the following mathematical operations to solve for the correct answer:

$$(2 + 3) + (2 \times 4) + \left(\frac{6+2}{2} \right) = \_\_\_\_\_\_\_\_\_\_$$

Solution:

a. Mathematical operations are typically performed going from left to right within an equation and within sets of parentheses.

b. Perform all math operations within the sets of parentheses first.

$2 + 3 = 5$
$2 \times 4 = 8$
$\frac{6+2}{2} = \frac{8}{2} = 4$ Note that the addition of 6 and 2 was performed prior to dividing by 2.

c. Perform all math operations outside of the parentheses. In this case, add from left to right.

$5 + 8 + 4 = 17$

Example:

Solve the following equation:

$$(4 - 2) + (3 \times 4) - (10 \div 5) - 6 = \_\_\_\_\_\_$$

Solution:

a. Perform math operations inside each set of parentheses.

$4 - 2 = 2$
$3 \times 4 = 12$

$10 \div 5 = 2$

b. Perform addition and subtraction operations from left to right.

c. The final answer is $2 + 12 - 2 - 6 = 6$

There may be cases where several operations will be performed within multiple sets of parentheses. In these cases you must perform all operations within the innermost set of parentheses and work outward. You must continue to observe the hierarchical rules through out the problem. Additional sets of parentheses may be indicated by brackets, [ ].

Example:

Solve the following equation:

$[2 \cdot (3 + 5) - 5 + 2] \times 3 =$ ______

Solution:

a. Perform operations in the innermost set of parentheses.

$3 + 5 = 8$

b. Rewriting the equation:

$[2 \cdot 8 - 5 + 2] \times 3 =$

c. Perform multiplication prior to addition and subtraction within the brackets.

$[16 - 5 + 2] \times 3 =$
$[11 + 2] \times 3 =$
$[13] \times 3 =$

d. Perform multiplication outside the brackets.

$13 \times 3 = 39$

Example:

Solve the following equation:

5 + [2 (3 + 1) - 1] x 2 = _____

Solution:

5 + [2 (4) - 1] x 2 =
5 + [8 - 1] x 2 =
5 + [7] x 2 =
5 + 14 = 19

Example:

Solve the following equation:

[(10 - 4) ÷ 3] + [4 x (5 - 3)] = _____

Solution:

[(6) ÷ 3] + [4 x (2)] =
[2] + [8] =
2 + 8 = 10

Summary

The important information from this chapter is summarized below.

Four Basic Arithmetic Operations Summary

This chapter reviewed using whole numbers to perform the operations of:

- Addition
- Subtraction
- Multiplication
- Division

While this chapter discussed the commutative and associative laws for whole numbers, it should be noted that these laws will also apply to the other types of numbers discussed in later chapters and modules of this course.

AVERAGES

This chapter covers the concept of averages and how to calculate the average of a given set of data.

EO 1.3 Given a set of numbers, CALCULATE the average value.

An average is the sum of a group of numbers or quantities divided by the number of numbers or quantities. Averages are helpful when summarizing or generalizing a condition resulting from different conditions. For example, when analyzing reactor power level, it may be helpful to use the average power for a day, a week, or a month. The average can be used as a generalization of the reactor power for the day, week, or month.

Average calculations involve the following steps:

Step 1: Add the individual numbers or quantities.
Step 2: Count the number of numbers or quantities.
Step 3: Divide the sum in Step 1 by the number in Step 2.

Example 1:

Find the average cost of a car, given the following list of prices.
$10,200; $11,300; $9,900; $12,000; $18,000; $7,600

Solution:

Step 1: 10200 + 11300 + 9900 + 12000 + 18000 + 7600 = 69000
Step 2: Total number of prices is 6
Step 3: Divide 69000 by 6. The result is 11500

Thus, the average price of the six cars is $11,500.

Example 2:

Find the average temperature if the following values were recorded: 600°F, 596°F, 597°F, 603°F

Solution:

Step 1: 600 + 596 + 597 + 603 = 2396
Step 2: The number of items is 4.
Step 3: 2396/4 = 599°F

Average Value

The summation symbol, $\sum$, introduced in the first chapter, is often used when dealing with the average value, $\bar{x}$.

Using the first example in this chapter, the average value could have been expressed in the following manner:

$$\bar{x}_{car} = \frac{\sum_{i=1}^{N} x_i}{N}$$

where:

$\bar{x}_{car}$	=	the average value (cost) of a car
x_i	=	each of the individual car prices
N	=	total number of cars

The right side of the above equation can then be rewritten.

$$\bar{x}_{car} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6}$$

substituting 10,200 for x_1, 11300 for x_2, 9,900 for x_3, etc.

$$\bar{x}_{car} = \frac{10{,}200 + 11{,}300 + 9{,}900 + 12{,}000 + 18{,}000 + 7600}{6}$$

$$\bar{x}_{car} = 11{,}500$$

Example:

If we were to apply the average value equation from above to the second example concerning temperature, how would it be written, and what would be the values for N_1, x_i?

Solution:

$$\bar{x}_{temp} = \frac{\sum_{i=1}^{4} x_i}{4}$$

$x_1 = 600$
$x_2 = 596$
$x_3 = 597$
$x_4 = 603$

$$\bar{x}_{temp} = \frac{x_1 + x_2 + x_3 + x_4}{4}$$

$$= \frac{600 + 596 + 597 + 603}{4}$$

$$= 599$$

Summary

The important information from this chapter is summarized below.

Averages Summary

Calculating the average of a set of numbers requires three steps:

1. Add the individual numbers or quantities.
2. Count the number of numbers or quantities added in previous step.
3. Divide the sum calculated in step 1 by the number in step 2.

FRACTIONS

This chapter covers the basic operations of addition, subtraction, multiplication, and division using fractions.

EO 1.4 APPLY one of the arithmetic operations of addition, subtraction, multiplication, and division using fractions.

A common fraction, such as $\frac{1}{3}$, consists of the numerator 1 and the denominator 3. It is referred to as a rational number describing the division of 1 by 3 (division of the numerator by the denominator).

Proper and Improper Fractions

There are two types of fractions: proper fractions and improper fractions. The value of the numerator and the denominator determines the type of fraction. If the numerator is less than the denominator, the fraction is less than one; this fraction is called a proper fraction. If the numerator is equal to or greater than the denominator, the fraction is called an improper fraction.

Example:

$\frac{3}{8}$ proper fraction

$\frac{8}{3}$ improper fraction

$\frac{3}{3}$ improper fraction

An improper fraction expressed as the sum of an integer and a proper fraction is called a mixed number.

To write an improper fraction as a mixed number, divide the numerator by the denominator, obtaining an integer part (quotient) plus a fractional part whose numerator is the remainder of the division.

Example:

$$\frac{22}{9} = 2 + \frac{4}{9} = 2\frac{4}{9}$$

Here, 9 can be divided into 22 two times, with $\frac{4}{9}$ left over or remaining.

Thus, the improper fraction $\frac{22}{9}$ is equivalent to the mixed number $2\frac{4}{9}$.

Every number may be expressed as a fraction or sum of fractions. A whole number is a fraction whose denominator is 1. Any fraction with the same numerator and denominator is equal to one.

Examples:

$$5 = \frac{5}{1}, \ \frac{10}{1} = 10, \ 1 = \frac{16}{16}, \ \frac{5}{5} = 1$$

Equivalent Fractions

An equivalent fraction is a fraction that is equal to another fraction.

Example:

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$$

A fraction can be changed into an equivalent fraction by multiplying or dividing the numerator and denominator by the same number.

Example:

$$\frac{2}{3} \cdot \frac{2}{2} = \frac{4}{6}$$ because $\frac{2}{2} = 1$, and 1 x any number = that number

A fraction may be reduced by dividing both the numerator and the denominator of a fraction by the same number.

Example:

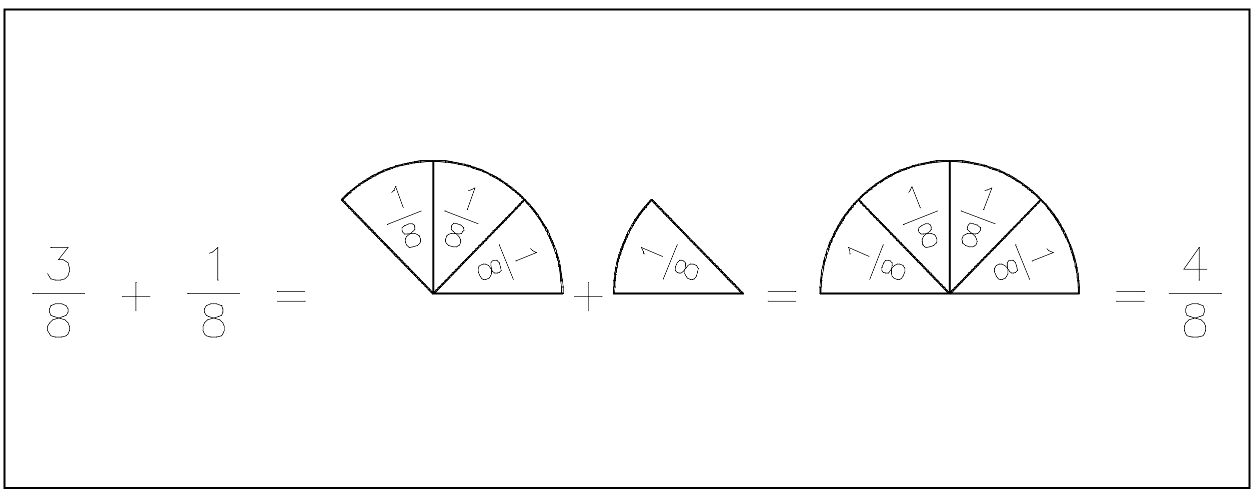
$$\frac{6}{8} = \frac{\frac{6}{2}}{\frac{8}{2}} = \frac{3}{4}$$

Addition and Subtraction of Fractions

When two or more fractions have the same denominator, they are said to have a common denominator. The rules for adding fractions with a common denominator will first be explored. Consider the example.

$$\frac{3}{8} + \frac{1}{8} = \underline{\qquad}$$

First of all, the fraction $\frac{3}{8}$ means three $\frac{1}{8}$ segments, i.e. $\frac{3}{8} = 3 \times \frac{1}{8}$. Looking at this as the addition of pie segments:

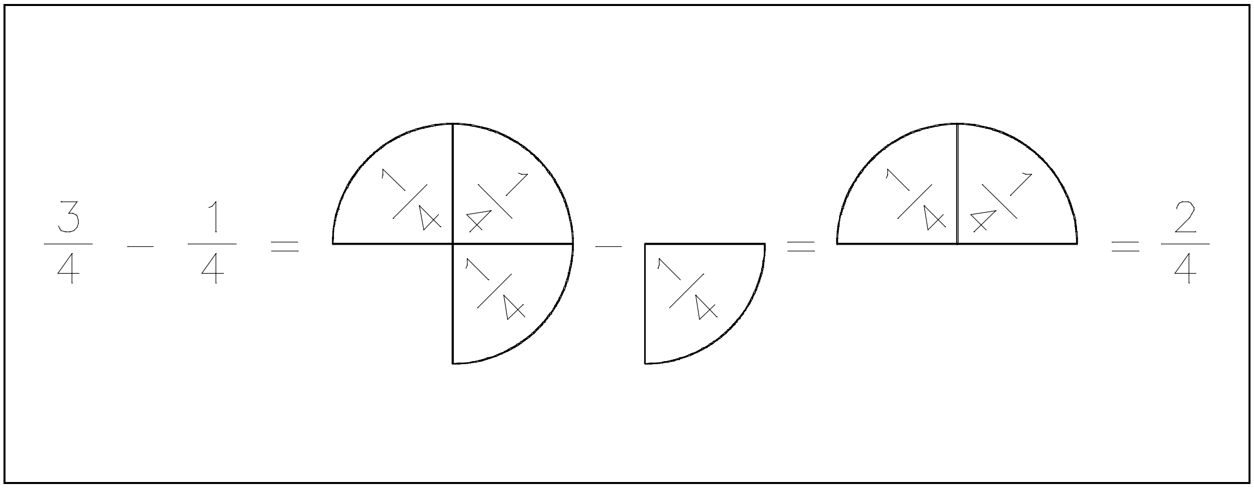


It is obvious that three of these segments $\left(\frac{1}{8}ths\right)$ plus one of these segments $\left(\frac{1}{8}ths\right)$ equal four of these segments $\left(\frac{1}{8}ths\right)$.

This graphic illustration can be done for any addition of fractions with common denominators. The sum of the fractions is obtained by adding the numerators and dividing this sum by the common denominator.

$$\frac{2}{6} + \frac{3}{6} + \frac{1}{6} = 2 \times \left(\frac{1}{6}\right) + 3 \times \left(\frac{1}{6}\right) + 1 \times \left(\frac{1}{6}\right) = 6 \times \frac{1}{6} = 1$$

Also, this general method applies to subtraction, for example,

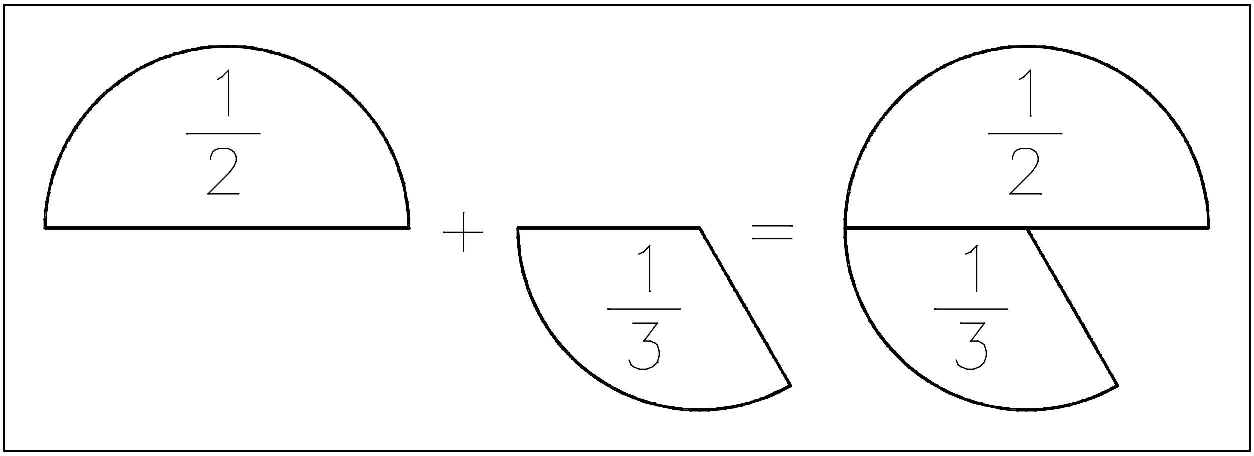


The general method of subtraction of fractions with common denominators is to subtract the numerators and place this difference over the common denominator.

$$\frac{5}{8} - \frac{2}{8} = 5 \times \left(\frac{1}{8}\right) - 2 \times \left(\frac{1}{8}\right) = (5-2) \times \left(\frac{1}{8}\right) = 3 \times \left(\frac{1}{8}\right) = \frac{3}{8}$$

When fractions do not have a common denominator, this method must be modified. For example, consider the problem:

$$\frac{1}{2} + \frac{1}{3} = ?$$

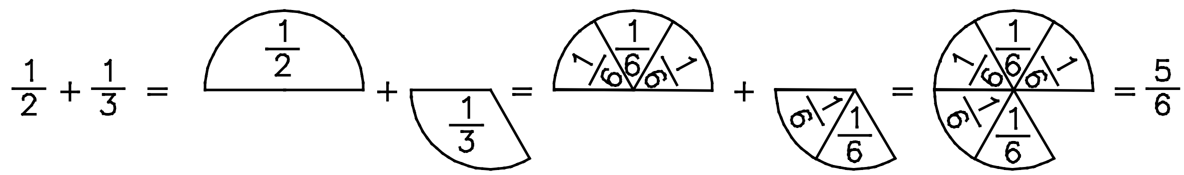


This presents a problem, the same problem one would have if he were asked to add 6 feet to 3 yards. In this case the entities (units) aren't equal, so the 6 feet are first converted to 2 yards and then they are added to 3 yards to give a total of 5 yards.

6 feet + 3 yards = 2 yards + 3 yards = 5 yards

Going back to the fraction addition example, then $\frac{1}{2}$ and $\frac{1}{3}$ must both be expressed in the same segments to be added. Without developing the general method, $\frac{1}{2}$ is $\frac{3}{6}$*ths*. Multiply $\frac{1}{2}$ by $\frac{3}{3}$ or (one) to give the equivalent fraction. Similarly, $\frac{1}{3}$ equals $\frac{2}{6}$.

Then,



The general method of adding or subtracting fractions which do not have a common denominator is to convert the individual fractions to equivalent fractions with a common denominator. These equally sized segments can then be added or subtracted.

The simplest method to calculate a common denominator is to multiply the denominators. This is obtained if each fraction is multiplied top and bottom by the denominator of the other fraction (and thus by one, giving an equivalent fraction).

$$\frac{1}{3} + \frac{8}{6} =$$

$$\frac{1}{3} \cdot \frac{6}{6} + \frac{8}{6} \cdot \frac{3}{3} =$$

$$\frac{6}{18} + \frac{24}{18} = \frac{30}{18}$$

For more than two fractions, each fraction is multiplied top and bottom by each of the other denominators. This method works for simple or small fractions. If the denominators are large or many fractions are to be added, this method is cumbersome.

Example:

$$\frac{105}{64} + \frac{15}{32} + \frac{1}{6} = \underline{\qquad}$$

would require the denominator to be equal to 64 x 32 x 6 = 12,288. This kind of number is very hard to use.

In the earlier example

$\frac{1}{3} + \frac{8}{6} =$ was shown to equal

$$\frac{6}{18} + \frac{24}{18} = \frac{30}{18}.$$

You notice that both 30 and 18 can be divided by 6; if this is done:

$$\frac{30 \div 6}{18 \div 6} = \frac{5}{3}$$

By doing this we arrive at a smaller and more useful number: $\frac{5}{3}$ takes the place of $\frac{30}{18}$.

The sum of two or more fractions reduced to its simplest form contains the smallest possible denominator common to both fractions. This denominator is called the least common denominator (LCD).

Example:

$$\frac{1}{3} + \frac{1}{6} + \frac{1}{8} = \underline{\qquad}$$

Using trial and error we can find that 24 is the LCD or smallest number that 3, 6, and 8 will all divide into evenly. Therefore, if each fraction is converted into 24^{ths}, the fractions can be added.

$$\frac{1}{3} \cdot \left(\frac{8}{8}\right) + \frac{1}{6} \cdot \left(\frac{4}{4}\right) + \frac{1}{8} \cdot \left(\frac{3}{3}\right) =$$

$$\frac{8}{24} + \frac{4}{24} + \frac{3}{24} = \frac{15}{24}$$

This is the simplest form the fraction can have. To eliminate the lengthy process of trial and error used in finding the LCD, you can reduce the denominators to their prime numbers.

Least Common Denominator Using Primes

A prime number is a whole number (integer) whose only factors are itself and one. The first prime numbers are:

1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ...

By dividing by primes, you can find that the primes of 105 are:

$$\frac{105}{3} = 35 \qquad \frac{35}{5} = 7 \qquad 7 = \text{a prime number, therefore, stop dividing.}$$

The primes of 105 are: 3, 5, 7

A systematic way of finding the prime factors of larger positive integers is illustrated below. The primes are tried in order, as factors, using each as many times as possible before going on to the next. The result in this case is:

$$\begin{aligned} 504 \quad &=(2)(252) \\ &=(2)(2)(126) \\ &=(2)(2)(2)(63) \\ &=(2)(2)(2)(3)(21) \\ &=(2)(2)(2)(3)(3)(7) \end{aligned}$$

To add several fractions with different denominators, follow these steps:

Step 1: Express denominators in prime factors.

Step 2: Determine the least common denominator by using all of the prime numbers from the largest denominator, and then include each prime number from the other denominators so that each denominator can be calculated from the list of primes contained in the LCD.

Step 3: Rewrite using the least common denominator.

Step 4: Add the fractions.

Example 1:

Add $\frac{1}{15}$ and $\frac{7}{10}$

Solution:

Step 1: Find primes of each denominator.

$$15 = 5 \times 3$$
$$10 = 5 \times 2$$

Step 2: In the example, 15 is the largest denominator, so use the 5 and the 3; now look at the second denominator's primes—the five already appears in the list, but the 2 does not, so use the 2.

$$5 \times 3 \times 2 = 30$$

Step 3: Rewrite with least common denominators.

$$\frac{1}{15} = \frac{2}{30}$$
$$\frac{7}{10} = \frac{21}{30}$$

Step 4: Add the new fractions.

$$\frac{2}{30} + \frac{21}{30} = \frac{23}{30}$$

Example 2:

Add $\frac{1}{7} + \frac{2}{3} + \frac{11}{12} + \frac{4}{6}$

Solution:

Step 1: Find primes of each denominator.

7 = 7 (already is a prime number)
3 = 3 (already is a prime number)
$12 = 2 \times 6 = 2 \times 2 \times 3$
$6 = 2 \times 3$

Step 2: 12 is the largest, so start with
$2 \times 2 \times 3$

Comparing this list to the others, the denominators of 3, 12, and 6 can all be calculated from the list, but 7 cannot be, so a 7 must be included in the list.

$$2 \times 2 \times 3 \times 7 = 84$$

Step 3: Rewrite the equation

$$\frac{1}{7} \cdot \frac{12}{12} + \frac{2}{3} \cdot \frac{28}{28} + \frac{11}{12} \cdot \frac{7}{7} + \frac{4}{6} \cdot \frac{14}{14} =$$

Step 4: Add

$$\frac{12}{84} + \frac{56}{84} + \frac{77}{84} + \frac{56}{84} = \frac{201}{84}$$

Addition and Subtraction

Denominators of fractions being added or subtracted must be the same.

The resulting sum or difference is then the sum or difference of the numerators of the fractions being added or subtracted.

Examples:

$$\frac{2}{3} + \frac{1}{3} = \frac{2+1}{3} = 1$$

$$\frac{4}{7} + \frac{1}{7} = \frac{4+1}{7} = \frac{5}{7}$$

$$\frac{4}{9} - \frac{2}{9} = \frac{4-2}{9} = \frac{2}{9}$$

$$\frac{8}{11} - \frac{2}{11} - \frac{5}{11} = \frac{8-2-5}{11} = \frac{1}{11}$$

Multiplication

The methods of multiplication of fractions differ from addition and subtraction. The operation of multiplication is performed on both the numerator and the denominator.

Step 1: Multiply the numerators.

Step 2: Multiply the denominators.

Step 3: Reduce fraction to lowest terms.

Example:

$$\frac{2}{3} \cdot \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$$

Multiplication of mixed numbers may be accomplished by changing the mixed number to an improper fraction and then multiplying the numerators and denominators.

Example:

$$1\frac{1}{2} \cdot \frac{3}{5} = \frac{3}{2} \cdot \frac{3}{5} = \frac{9}{10}$$

Division

The division of fractions can be performed by two methods. The first method employs the basic concept of multiplying by 1.

Example:

$$\frac{\left(\frac{4}{5}\right)}{\left(\frac{2}{9}\right)} = \underline{\qquad}$$

Solution:

Step 1: Multiply by $\frac{\left(\frac{9}{2}\right)}{\left(\frac{9}{2}\right)}$, which is the same as multiplying by 1.

$$\frac{\left(\frac{4}{5}\right)}{\left(\frac{2}{9}\right)} \cdot \frac{\left(\frac{9}{2}\right)}{\left(\frac{9}{2}\right)} = \_\_\_\_\_$$

Step 2: Looking at the two division fractions we see that $\frac{2}{9} \cdot \frac{9}{2} = 1$. This leaves us with the following.

$$\frac{\frac{4}{5} \cdot \frac{9}{2}}{1} = \frac{4}{5} \cdot \frac{9}{2}$$

Step 3: Multiply numerators and denominators.

$$\frac{4}{5} \cdot \frac{9}{2} = \frac{36}{10}$$

Example:

$$\frac{\left(\frac{3}{8}\right)}{\left(\frac{6}{7}\right)} = \_\_\_\_\_$$

Solution:

Step 1: Multiply by $\frac{\frac{7}{6}}{\frac{7}{6}}$.

$$\frac{\frac{3}{8} \cdot \frac{7}{6}}{\frac{6}{7} \cdot \frac{7}{6}} =$$

Step 2: Multiplication of division fractions equals 1.

$$\frac{\frac{3}{8} \cdot \frac{7}{6}}{1} =$$

Step 3: Multiplication of numerators and denominators yields:

$$\frac{3}{8} \cdot \frac{7}{6} = \frac{21}{48}$$

The second method for dividing fractions is really a short cut to the first method. When dividing one fraction by another, first invert the divisor fraction and then multiply.

Example:

$$\frac{\left(\frac{4}{5}\right)}{\left(\frac{2}{9}\right)} =$$

Solution:

Step 1: Invert the divisor fraction $\left(\frac{2}{9}\right)$ to $\left(\frac{9}{2}\right)$.

Step 2: Multiply the dividend fraction, $\left(\frac{4}{5}\right)$, by the inverted fraction $\left(\frac{9}{2}\right)$.

$$\left(\frac{4}{5}\right) \cdot \left(\frac{9}{2}\right) = \frac{36}{10}$$

Step 3: Reduce fraction to lowest terms.

$$\frac{\left(\frac{36}{2}\right)}{\left(\frac{10}{2}\right)} = \frac{18}{5} = 3\frac{3}{5}$$

Division of mixed numbers may be accomplished by changing the mixed number into an improper fraction (*a*/*b*), inverting the divisor, and proceeding as in multiplication.

Invert the divisor fraction and then follow the rule for multiplication.

Example:

$$\frac{1\frac{2}{3}}{\frac{3}{7}} = \frac{5}{3} \cdot \frac{7}{3} = \frac{35}{9} = 3\frac{8}{9}$$

Summary

The important information from this chapter is summarized below.

Fractions Summary

Denominator - bottom number in a fraction

Numerator - top number in a fraction

Proper fraction - numerator is less than denominator

Improper fraction - numerator is greater than or equal to denominator

Mixed number - sum of an integer and a proper fraction

Fractions, like whole numbers can be:

a. Added

b. Subtracted

c. Multiplied

d. Divided

DECIMALS

This chapter covers the processes of addition, subtraction, multiplication, and division of numbers in decimal form.

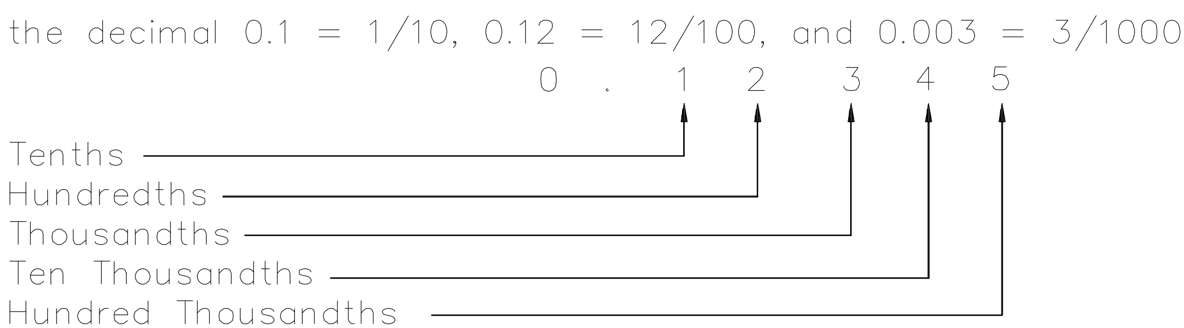
EO 1.5 APPLY one of the arithmetic operations of addition, subtraction, multiplication, and division of fractions by conversion to decimal form using a calculator.

EO 1.6 APPLY one of the arithmetic operations of addition, subtraction, multiplication, and division using decimals.

When using numbers, the operator will use whole numbers at times and decimal numbers at other times. A decimal number is a number that is given in decimal form, such as 15.25. The decimal portion is equivalent to a certain "fraction-of-one," thus allowing values between integer numbers to be expressed.

A decimal is a linear array of integers that represents a fraction. Every decimal place indicates a multiple of a power of 10.

Example:



Fraction to Decimal Conversion

In the process of converting a fraction to a decimal, we must perform the operation of division that the fraction represents.

Example:

Convert $\frac{3}{4}$ to a decimal.

Solution:

The fraction $\frac{3}{4}$ represents 3 divided by 4. To put this into decimal form, we first divide 3 by 4. Add a decimal point and zeros to carry out this division.

$$
\begin{array}{r}
0.75 \\
4\overline{)\,3.00} \\
\underline{2\,8} \\
20 \\
\underline{20} \\
0
\end{array}
$$

Example:

Convert $\frac{1}{3}$ to a decimal.

Solution:

$$
\begin{array}{r}
0.33\overline{3} \\
3\overline{)\,1.000} \\
\underline{9} \\
10 \\
\underline{9} \\
10 \\
\underline{9} \\
1
\end{array}
$$

In the above example we see that no matter how many zeros we add, there will always be a remainder of 1. This is called a repeating decimal. A repeating decimal is indicated by a dash over the last number to the right of the decimal point. So, $\frac{1}{3} = 0.33\overline{3}$. The bar is placed over the repeating portion. For a repeating single digit, the bar is placed over only a single digit. For a repeating sequence of digits, the bar is placed over the whole sequence of digits.

Decimal to Fraction Conversion

The process of decimal to fraction conversion involves the use of the fundamental rule of fractions; the fraction should be written in its lowest terms. The following examples demonstrate how to convert decimals to fractions.

Example 1:

Convert 0.65 to a fraction.

Solution:

Step 1: Note the number of place positions to the right of the decimal point. In this example, 0.65 is 65 hundredths, which is two places to the right of the decimal point.

$$\frac{65}{100}$$

Step 2: Although we have now converted the decimal into a fraction, the fraction is not in its lowest terms. To reduce the new fraction into its lowest or simplest terms, both the numerator and the denominator must be broken down into primes.

$$\frac{65}{100} = \frac{5 \cdot 13}{5 \cdot 20} = \frac{5 \cdot 13}{5 \cdot 4 \cdot 5} = \frac{5 \cdot 13}{5 \cdot 2 \cdot 2 \cdot 5}$$

Note that we can cancel one set of 5s, because $\frac{5}{5} = 1$.

This gives

$$\frac{65}{100} = \frac{13}{20}$$

and this is the simplest form of this fraction.

Example 2:

Convert 18.82 to a mixed number.

Solution:

Step 1: 18.82 is 18 and 82 hundredths.

$$18.82 = 18\frac{82}{100}$$

Step 2: Reduce $\frac{82}{100}$ to its simplest form $= \frac{41}{50}$

$$\frac{82}{100} = \frac{2 \cdot 41}{2 \cdot 50} = \frac{2 \cdot 41}{2 \cdot 2 \cdot 25} = \frac{2 \cdot 41}{2 \cdot 2 \cdot 5 \cdot 5} = \frac{41}{2 \cdot 5 \cdot 5} = \frac{41}{50}$$

The answer is $18\frac{41}{50}$.

Example 3:

Convert 1.73 to a fraction.

Solution:

Step 1: $1.73 = 1\frac{73}{100}$

Step 2: 73 = 73 x 1
100 = 2 x 2 x 5 x 5
There are no common factors between 73 and 100, so it cannot be reduced.

$$1\frac{73}{100}$$

Example 4:

Convert 0.333 to a fraction.

Solution:

Step 1: $0.333 = \frac{333}{1000}$

Step 2: There are no common factors between 333 and 1000, so it is already in its simplest form.

Addition and Subtraction of Decimals

When adding or subtracting decimals, each number must be placed to align the decimal points. When necessary, zeros are used as place holders to make this possible. Then the operation of addition or subtraction is performed.

Example:

0.423 + 1.562 + 0.0736 + 0.2 = ____

Solution:

Align decimal points

$$\begin{array}{r} 0.4230 \\ +\ 1.5620 \\ +\ 0.0736 \\ 0.2000 \\ \hline 2.2586 \end{array}$$

Example:

0.832 - 0.0357 = ____

Solution:

$$\begin{array}{r} 0.8320 \\ -\ 0.0357 \\ \hline 0.7963 \end{array}$$

Multiplying Decimals

When multiplying decimals, the decimal points do not have to be aligned. Rather, it is important to accurately position the decimal point in the product. To position the decimal in the product, the total number of digits to the right of the decimals in the numbers being multiplied must be equal to the number of digits to the right of the decimal in the product. This is best illustrated in the following examples:

Step 1: Multiply numbers without inserting decimal in the products.

Step 2: Sum the number of digits to the right of the decimal in all of the numbers being multiplied.

Step 3: Position the decimal in the product so the number of digits to the right of the decimal equals the total number of digits to the right of the decimal in the numbers multiplied (from Step 2).

Example:

0.056 x 0.032 = ____

Solution:

```
   0.056
  0.032
  ------
     112
    168
 --------
 0.001792
```

NOTE: Since 0.056 has three digits to the right of the decimal point, and 0.032 has three digits to the right of the decimal point, six digits must be to the right of the decimal point in the product. To have six digits in the product, zeros are inserted to the left of the computed digits.

To multiply a decimal by 10, move the decimal point one position to the right.

Example: 0.45 x 10 = 4.5. Similarly, when multiplying a decimal by 100, 1000, and 10,000, move the decimal point to the right the same number of zeros that are in the multiplier.

Example:

0.45 x 100 = 45
0.45 x 1000 = 450
0.45 x 10,000 = 4500

The reverse is true when multiplying by fractions of 10.

0.45 x 0.1 = 0.045
0.45 x 0.01 = 0.0045
0.45 x 0.001 = 0.00045
0.45 x 0.0001 = 0.000045

Dividing Decimals

When solving problems involving division of decimals, the following procedure should be applied.

Step 1: Write out the division problem.

Step 2: Move the decimal in the divisor to the right.

Step 3: Move the decimal in the dividend the same number of places to the right. Add zeros after the decimal in the dividend if necessary.

Step 4: Place the decimal point in the quotient directly above the decimal in the dividend.

Step 5: Divide the numbers.

Example:

$$3.00 \div 0.06 = \underline{\quad\quad}$$

Solution:

Step 1: $.06\overline{)3.00}$

Step 2: $.06.\overline{)\ 3.00}$

Step 3: $6.\overline{)\ 300.}$

Step 4: $6.\overline{)\ 300.}$

Step 5:
$$\begin{array}{r} 50. \\ 6\overline{)\ 300.} \\ \underline{30\ \ } \\ 00 \end{array}$$

Rounding Off

When there is a remainder in division, the remainder may be written as a fraction or rounded off. When rounding off, the following rules should be applied:

Step 1: Observe the digit to the right of the digit being rounded off.

Step 2: If it is less than 5, drop the digit.
If the digit is 5 or higher, add 1 to the digit being rounded off.

Step 3: Write the new rounded number.

Example:

Round off the following number to two decimal places.

3.473

Solution:

Step 1: 3 is the number to the right of the 2^{nd} decimal place.

Step 2: 3 is less than 5, so drop the digit.

Step 3: 3.47 is the number rounded to two decimal places.

Example:

Round off the following number to two decimal places.

6.238

Solution:

Step 1: 8 is the number to the right of the 2^{nd} decimal place.

Step 2: 8 is greater than 5, so drop the 8 and add one to the number in the second decimal place (3 + 1 = 4).

Step 3: 6.24 is the number rounded to two decimal places.

Example:

Round off the following number to two decimal places.

6.2385

Solution:

Step 1: 8 is the number to the right of the 2^{nd} decimal place.

Step 2: 8 is greater than 5, so drop the 8 and add one to number in the second decimal place (3 + 1 = 4).

Step 3: 6.24 is the number rounded to two decimal places.

Example:

Round off the following number to three decimal places.

6.2385

Solution:

Step 1: 5 is the number to the right of the 3rd decimal place.

Step 2: 5 is equal to 5, so drop the 5 and add one to the number in the third decimal place ($8 + 1 = 9$).

Step 3: 6.239 is the number rounded to three decimal places.

Example:

Divide 2.25 by 6 and round off the answer to 1 decimal place.

$$\frac{2.25}{6} = 0.375$$

Solution:

Step 1: 7 is the number to the right of the 1st decimal place.

Step 2: 7 is greater than 5, so drop the 7 and add one to the number in the first decimal place ($3 + 1 = 4$).

Step 3: 0.4 is .375 rounded to 1 decimal place.

Summary

The important information from this chapter is summarized below.

Decimals Summary

When using the decimal process:

Convert fractions to decimals by dividing the numerator by the denominator.

Convert decimals to fractions by writing the decimal in fraction format and reducing.

Align decimal points when adding or subtracting decimals.

Before dividing decimals, move the decimal in the divisor and dividend to the right by the same number of places.

When rounding, numbers less than 5 are dropped, and numbers 5 or greater increase the number immediately to the left by one.

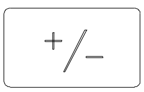
SIGNED NUMBERS

This chapter covers the processes of addition, subtraction, division, and multiplication of signed numbers.

EO 1.7 APPLY one of the arithmetic operations of addition, subtraction, multiplication, and division using signed numbers.

Calculator Usage, Special Keys

Change Sign key



Pressing this key changes the sign of the number in the display. To enter a negative number, the number is entered as a positive number and then the change sign key is pressed to convert it to a negative. The display will show a "-" in front of the number.

Addition

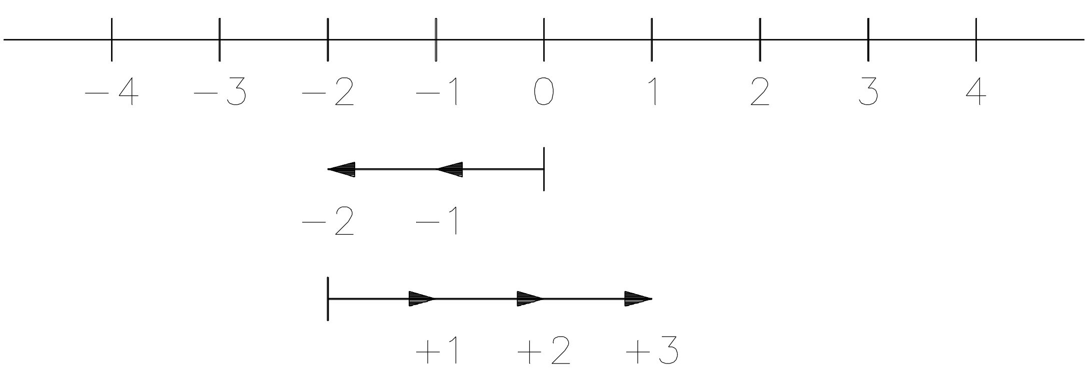
Addition of signed numbers may be performed in any order. Begin with one number and count to the right if the other number is positive or count to the left if the other number is negative.

Example:

$$-2 + 3 = 0 - 2 + 3$$

Solution:

Begin with –2 and count 3 whole numbers to the right.

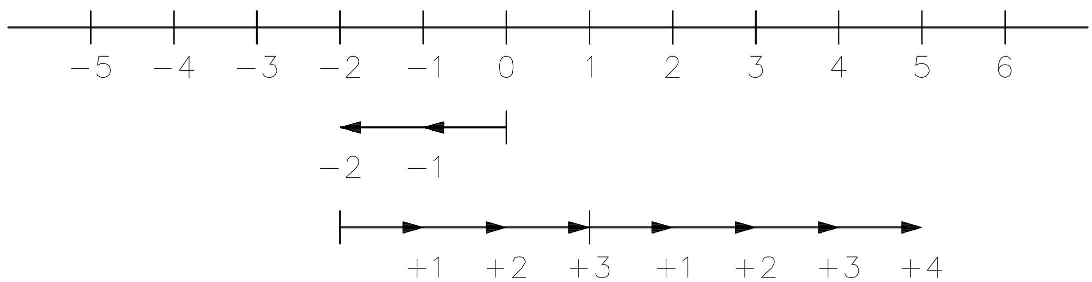


Therefore: -2 + 3 = 1

Example:

(-2) + 3 + 4 = 0 - 2 + 3 + 4

Solution:



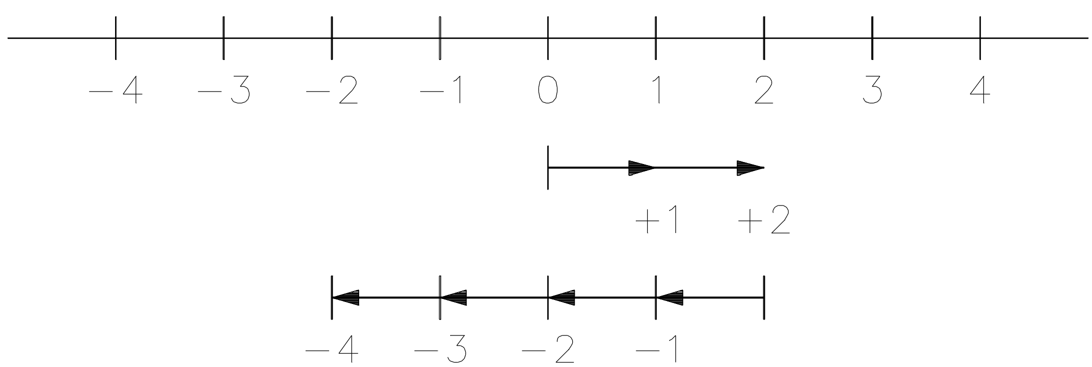
Therefore: (-2) + 3 + 4 = 5

Example:

(2) + (–4) = ____

Solution:

Begin with 2 and count 4 whole numbers to the left.



Therefore: (2) + (–4) = –2

Adding numbers with unlike signs may be accomplished by combining all positive numbers, then all negative numbers, and then subtracting.

Example:

$$10 + (-5) + 8 + (-7) + 5 + (-18) = \_\_\_\_\_$$

Solution:

$$+10 - 5 + 8 - 7 + 5 - 18 =$$

$$+10 + 8 + 5 - 18 - 7 - 5 =$$

$$+23 - 30 = -7$$

Subtraction

Subtraction of signed numbers may be regarded as the addition of numbers of the opposite signs. To subtract signed numbers, reverse the sign of the subtrahend (the second number) and add.

For example, one could treat his incomes for a given month as positive numbers and his bills as negative numbers. The difference of the two is his increase in cash balance. Suppose he buys a window for $40. This gives a bill of $40 and adds as negative $40 to his cash balance. Now suppose he returns this window to the store and the manager tears up his bill, subtracting the -$40. This is equivalent of adding +$40 to his cash balance.

Example:

$$a - b \quad = \quad a + (-b)$$

Solution:

$$(+3) - (+5) \quad = \quad (+3) + (-5) \quad = -2$$

$$(-4) - (-1) \quad = \quad (-4) + (+1) \quad = -3$$

$$(-5) - (+8) \quad = \quad (-5) + (-8) \quad = -13$$

$$(+7) - (-2) \quad = \quad (+7) + (+2) \quad = +9$$

Multiplication

Multiplication of signed numbers may be performed by using the following rules:

The product of any two numbers with like signs is positive:
$(+)(+) = (+)$ or $(-)(-) = (+)$.

The product of any two numbers with unlike signs is negative:
$(+)(-) = (-)$ or $(-)(+) = (-)$.

The product is negative if there is an odd number of negatives.
The product is positive if there is an even number of negatives.

Example:

$(+3)(+3) = +9$
$(-2)\ (+4) = -8$
$(-1)\ (-2)\ (+1)\ (-2) = -4$
$(-2)\ (+2)\ (+2)\ (-2) = +16$

Zero times any number equals zero.
Multiplying by -1 is the equivalent of changing the sign.

Division

Division of signed numbers may be performed using the following rules:

Rule 1: The quotient of any two numbers with like signs is positive:
$(+)/(+) = (+)$ or $(-)/(-) = (+)$

Rule 2: The quotient of any two numbers with unlike signs is negative:
$(+)/(-) = (-)$ or $(-)/(+) = (-)$

Rule 3: Zero divided by any number not equal to zero is zero.

Examples:

a) $\frac{0}{-5} = 0$ Apply rule 3.

b) $\frac{-3}{-1} = +3$ Apply rule 1.

c) $\frac{-4}{+2} = -2$ Apply rule 2.

Summary

The important information from this chapter is summarized below.

Signed Numbers Summary

When using signed numbers:

Adding a negative number is the same as subtracting a positive number.

Subtracting a negative number is the same as adding a positive number.

A product is negative if there is an odd number of negatives.

A product is positive if there is an even number of negatives.

Division of two numbers with like signs results in a positive answer.

Division of two numbers with unlike signs results in a negative answer.

SIGNIFICANT DIGITS

This chapter presents the concept of significant digits and the application of significant digits in a calculation.

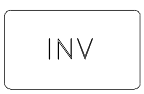
EO 1.8 DETERMINE the number of significant digits in a given number.

EO 1.9 Given a formula, CALCULATE the answer with the appropriate number of significant digits.

Calculator Usage, Special Keys

Most calculators can be set up to display a fixed number of decimal places. In doing so, the calculator continues to perform all of its internal calculations using its maximum number of places, but rounds the displayed number to the specified number of places.

INV key



To fix the decimal place press the INV key and the number of the decimal places desired. For example, to display 2 decimal places, enter INV 2.

Significant Digits

When numbers are used to represent a measured physical quantity, there is uncertainty associated with them. In performing arithmetic operations with these numbers, this uncertainty must be taken into account. For example, an automobile odometer measures distance to the nearest 1/10 of a mile. How can a distance measured on an odometer be added to a distance measured by a survey which is known to be exact to the nearest 1/1000 of a mile? In order to take this uncertainty into account, we have to realize that we can be only as precise as the least precise number. Therefore, the number of significant digits must be determined.

Suppose the example above is used, and one adds 3.872 miles determined by survey to 2.2 miles obtained from an automobile odometer. This would sum to 3.872 + 2.2 = 6.072 miles, but the last two digits are not reliable. Thus the answer is rounded to 6.1 miles. Since all we know about the 2.2 miles is that it is more than 2.1 and less than 2.3, we certainly don't know the sum to any better accuracy. A single digit to the right is written to denote this accuracy.

Both the precision of numbers and the number of significant digits they contain must be considered in performing arithmetic operations using numbers which represent measurement. To determine the number of significant digits, the following rules must be applied:

Rule 1: The left-most non-zero digit is called the most significant digit.

Rule 2: The right-most non-zero digit is called the least significant digit except when there is a decimal point in the number, in which case the right-most digit, even if it is zero, is called the least significant digit.

Rule 3: The number of significant digits is then determined by counting the digits from the least significant to the most significant.

Example:

In the number 3270, 3 is the most significant digit, and 7 is the least significant digit.

Example:

In the number 27.620, 2 is the most significant digit, and 0 is the least significant digit.

When adding or subtracting numbers which represent measurements, the right-most significant digit in the sum is in the same position as the left-most least significant digit in the numbers added or subtracted.

Example:

15.62 psig + 12.3 psig = 27.9 psig

Example:

401.1 + 50 = 450

Example:

401.1 + 50.0 = 451.1

When multiplying or dividing numbers that represent measurements, the product or quotient has the same number of significant digits as the multiplied or divided number with the least number of significant digits.

Example:

3.25 inches x 2.5 inches = 8.1 inches squared

Summary

The important information from this chapter is summarized below.

Significant Digits Summary

Significant digits are determined by counting the number of digits from the most significant digit to the least significant digit.

When adding or subtracting numbers which represent measurements, the right-most significant digit in the sum is in the same position as the left-most significant digit in the numbers added or subtracted.

When multiplying or dividing numbers that represent measurements, the product or quotient has the same number of significant digits as the multiplied or divided number with the least number of significant digits.

PERCENTAGES

This chapter covers the conversion between percents, decimals, and fractions.

EO 1.10 CONVERT between percents, decimals, and fractions.

EO 1.11 CALCULATE the percent differential.

A special application of proper fractions is the use of percentage. When speaking of a 30% raise in pay, one is actually indicating a fractional part of a whole, 30/100. The word percent means "hundredth;" thus, 30% is based on the whole value being 100%. However, to perform arithmetic operations, the 30% expression is represented as a decimal equivalent (0.30) rather than using the % form.

Calculator Usage, Special Keys

Percent Key

%

When pressed, the percent key divides the displayed number by 100.

Changing Decimals to Percent

Any number written as a decimal may be written as a percent. To write a decimal as a percent, multiply the decimal by 100, and add the percent symbol.

Example:

Change 0.35 to percent.
0.35 x 100 = 35%

Example:

Change 0.0125 to percent.
0.0125 x 100 = 1.25%

Example:

Change 2.7 to percent.
2.7 x 100 = 270%

Changing Common Fractions and Whole Numbers to Percent

When changing common fractions to percent, convert the fraction to a decimal, then multiply by 100 and add the percent symbol.

Example:

Change $\frac{3}{5}$ to a percent
0.6 x 100 = 60%

When changing a whole number to a percent, multiply by 100 and add the percent symbol.

Example:

Change 10 to percent
10 x 100 = 1000%

Percents are usually 100% or less. Percents are most often used to describe a fraction, but can be used to show values greater than 1(100%). Examples are 110%, 200%, etc.

Changing a Percent to a Decimal

Any number written as a percent may be written as a decimal. To change a percent to a decimal, drop the percent symbol and divide by 100.

Example:

Express 33.5% in decimal form.

$$\frac{33.5}{100} = 0.335$$

Express 3.35% in decimal form.

$$\frac{3.35}{100} = 0.0335$$

Express 1200% in decimal form.

$$\frac{1200}{100} = 12$$

Percent Differential

Percent differentials are used to provide a means of comparing changes in quantities or amounts. Percent differentials express the relationship between some initial condition and another specified condition.

The method of calculating percent differential involves the following:

Step 1: Subtract the original value from the present value.

Step 2: Divide by the original value.

Step 3: Multiply by 100.

Step 4: Add the percent symbol (%).

Example:

A tank initially contains 50 gallons of water. Five gallons are drained out. By what percent is the amount of water in the tank reduced?

Solution:

Step 1: The difference between initial and final is given in the problem: 5 gallons.

Step 2: $\frac{5}{50} = 0.1$

Step 3: 0.1 x 100 = 10% Five gallons represents 10% of the original 50 gals that were in the tank.

Ratio

Two numbers may be compared by expressing the relative size as the quotient of one number divided by the other and is called a **ratio**. Ratios are simplified fractions written with a colon (:) instead of a division bar or slash.

Example:

One day Eric paid $700 for a stereo and Scott paid $600 for the same stereo. Compare the amount that Eric paid to the amount that Scott paid, using ratios.

Solution:

Step 1: Divide the numbers to be compared. In this example the amount paid by Scott is being compared to the amount paid by Eric. The amount paid by Eric is divided by the amount paid by Scott = $\frac{700}{600}$.

Step 2: Simplifying this expression, both 700 and 600 can be divided by 100.

Step 3: Expressing this fraction as a ratio:

$$\frac{\text{Eric's price}}{\text{Scott's price}} = \frac{7}{6}$$ or Eric's price : Scott's price = 7:6

Example:

If one yard equals three feet, what is the ratio of yards to feet?

Solution:

Step 1: 1 yd./ 3 ft.

Step 2: $\frac{1}{3}$ is already in simplest terms

Step 3: $\frac{\text{yards}}{\text{feet}} = \frac{1}{3}$ or yards : feet = 1:3

Summary

Pertinent information concerning percentages and ratios is summarized below.

Percentages and Ratios Summary

Change decimals to percents by multiplying by 100 and adding the percent symbol.

Change fractions to percents by first changing the fraction into a decimal. Then change the decimal to a percent.

Compute percent differential by dividing the difference by the original value, multiplying by 100, and adding the percent symbol.

Ratios are fractions written with a colon instead of a division bar or slash.

EXPONENTS

This chapter covers the addition, subtraction, multiplication, and division of numbers with exponents.

EO 1.12 APPLY one of the arithmetic operations of addition, subtraction, multiplication, and division using exponential numbers.

Calculator Usage, Special Keys

Exponent key

$\boxed{y^x}$

Raising a number to an exponent requires the y^x key to be pressed twice. First, the base number is entered and the y^x key is pressed; this enters the base number (y). Next, the exponent number is pressed and the y^x key is pressed; this enters the exponent and tells the calculator to complete the calculation. The calculator will display the value.

x squared key

$\boxed{x^2}$

Pressing this key squares the displayed number. This key will save time over using the y^x key.

Exponents

The product a x a x a x a can be written as a^4, where 4 is called the exponent of a or power to which a is raised. In this notation, a is often called the base.

Examples:

$$a^4 = a \cdot a \cdot a \cdot a$$
$$5^3 = 5 \cdot 5 \cdot 5$$
$$(a + b)^5 = (a+b) \cdot (a+b) \cdot (a+b) \cdot (a+b) \cdot (a+b)$$

When an exponent is not written, it is assumed to be 1. For example, $a^1 = a$. An exponent applies only to the quantity immediately to the left and below it. For example, in $3 + (-2)^3$ the base is -2, but in $3 - 2^3$ the base is 2.

Basic Rules for Exponents

The following rules are applied to exponents.

Rule 1: To multiply numbers with the same base, add the exponents and keep the base the same.

$$a^m a^n = a^{m+n}$$

Example:

$2^2 \text{ x } 2^3 = (2 \text{ x } 2) \text{ x } (2 \text{ x } 2 \text{ x } 2) = 2 \text{ x } 2 \text{ x } 2 \text{ x } 2 \text{ x } 2 = 2^5$

Rule 2: When raising a power of a number to a power, multiply the exponents and keep the base the same.

$$(a^m)^n = a^{mn}$$

Example:

$(a^2)^3 = (a \text{ x } a) \text{ x } (a \text{ x } a) \text{ x } (a \text{ x } a) = a^6$

that is, you multiply (a x a) three times. Similarly, for $(a^m)^n$, one multiplies (a^m) n times. There are m values of a in each parenthesis multiplied by n parenthesis or m x n values of a to multiply.

Thus, $(a^m)^n = a^{mn}$

Rule 3: When dividing two exponential numbers, subtract the powers.

$$\frac{a^m}{a^n} = a^{m-n}$$

Example:

$$\frac{a^5}{a^2} = \frac{a \times a \times a \times a \times a}{a \times a} = \frac{\cancel{a}}{\cancel{a}} \times \frac{\cancel{a}}{\cancel{a}} \times a \times a \times a = a^3$$

Rule 4: Any exponential number divided by itself is equal to one.

$$\frac{a^n}{a^n} = 1$$

Rule 5: To raise a product to a power, raise each factor to that power.

$$(ab)^n = a^n b^n$$

This arises from the associative law for multiplication, that is, order of multiplication does not alter the product.

Example:

$(ab)^2 = (a \text{ x } b) \text{ x } (a \text{ x } b) = (a \text{ x } a)\ (b \text{ x } b) = a^2 \text{ x } b^2$

If doubt exists in the student's mind, try multiplying $(2 \text{ x } 3)^2$ out in different orders. All orders will yield 36.

Rule 6: To raise a quotient to a power, raise both the numerator and denominator to that power.

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Example:

To demonstrate this, consider $\left(\frac{3}{2}\right)^2 = 1.5^2 = 2.25 = 2\frac{1}{4} = \frac{9}{4}$

But $\frac{3^2}{2^2} = \frac{9}{4}$, the same value.

Zero Exponents

Using the rule for exponents (Rule 4) to evaluate a^n/a^n, then

$$\frac{a^n}{a^n} = 1$$

This interpretation is consistent with the rule $a^n/a^n = a^{n-n} = a^0$. Therefore, $a^0 = 1$ when a is not equal to 0. Any number to the zero power equals one.

Example:

$3^0 = 1$
$(b^2+2)^0 = 1$

Negative Exponents

The rules for positive exponents apply to negative exponents. a^{-n} is defined as follows:

$$a^{-n} = a^{-n} = \frac{1}{a^n}$$

$$a^n = \frac{1}{a^{-n}}$$

For example, $a^5/a^2 = a^{5-2}$ as shown earlier. If $\frac{1}{a^2}$ is written as a^{-2}, and the rules for multiplication are applied to this, $a^5 \text{ x } a^{-2} = a^{5-2} = a^3$. Thus, writing $\frac{1}{a^n}$ as a^{-n} and applying the rules for multiplication yields the same results as $\frac{1}{a^n}$ and applying the rules of division.

Examples:

$$c^{-2} = \frac{1}{c^2}$$

$$x^3 = \frac{1}{x^{-3}}$$

Fractional Exponents

Fractional exponents are defined as follows, $a^{\frac{1}{m}} \equiv \sqrt[n]{a}$. This permits manipulations with numbers with fractional exponents to be treated using the laws expressed earlier for integers. For example,

$$8^{\frac{1}{3}} \equiv \sqrt[3]{8} = 2 \qquad \text{since } 2 \times 2 \times 2 = 8$$

Taking the statement $8^{\frac{1}{3}} = 2$ and cubing both sides, $\left(8^{\frac{1}{3}}\right)^3 = 2^3$. But $(a^m)^n = a^{m \times n}$ so $\left(8^{\frac{1}{3}}\right)^3 = 8^1 = 8$ which agrees with $2^3 = 8$ for the right-hand side of the equality.

A number such as $8^{\frac{2}{3}}$ can be written $\left(8^{\frac{1}{3}}\right)^2 = 2^2 = 4$ or alternately as $\left(8^2\right)^{\frac{1}{3}} = (64)^{\frac{1}{3}} = 4$ since 4 x 4 x 4 = 64; that is, 4 is the cube root of 64.

Examples:

$$\left(a^{\frac{1}{3}}\right) \cdot \left(a^{\frac{2}{3}}\right) = a^{\left(\frac{1}{3} + \frac{2}{3}\right)} = a^1 = a$$

$$\frac{b^{\frac{1}{4}}}{b^{\frac{1}{2}}} = b^{\left(\frac{1}{4} - \frac{1}{2}\right)} = b^{\frac{-1}{2}} = \frac{1}{b^2}$$

$$\left(d^{\frac{1}{3}}\right)^9 = d^{\left(\frac{1}{3} \times 9\right)} = d^3$$

Summary

Pertinent information concerning exponents is summarized below.

Exponents Summary

$\text{Base}^{\text{Exponent}} = \text{Product}$

Rule 1: To multiply numbers with the same base, add the exponents and keep the base the same.

$$a^m a^n = a^{m+n}$$

Rule 2: When raising a power of a number to a power, multiply the exponents and keep the base the same.

$$(a^m)^n = a^{mn}$$

Rule 3: When dividing two exponential numbers, subtract the powers.

$$a^m/a^n = a^{m-n}$$

Rule 4: Any exponential number divided by itself is equal to one.

$$a^n/a^n = 1$$

Rule 5: To raise a product to a power, raise each factor to that power.

$$(ab)^n = a^n b^n$$

Rule 6: To raise a quotient to a power, raise both the numerator and denominator to that power.

$$(a/b)^n = a^n/b^n$$

Any number to the zero power equals one.

The rules for positive exponents apply to negative exponents.

The rules for integer exponents apply to fractional exponents.

SCIENTIFIC NOTATION

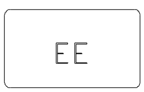
This chapter covers the addition, subtraction, multiplication, and division of numbers in scientific notation.

EO 1.13 Given the data, CONVERT integers into scientific notation and scientific notation into integers.

EO 1.14 APPLY one of the arithmetic operations of addition, subtraction, multiplication, and division to numbers using scientific notation.

Calculator Usage

Scientific Notation key



If pressed after a number is entered on the display, the EE key will convert the number into scientific notation. If a number is to be entered in scientific notation into the calculator, pressing the EE key tells the calculator the next entered numbers are the exponential values.

Scientists, engineers, operators, and technicians use scientific notation when working with very large and very small numbers. The speed of light is 29,900,000,000 centimeters per second; the mass of an electron is 0.000549 atomic mass units. It is easier to express these numbers in a shorter way called scientific notation, thus avoiding the writing of many zeros and transposition errors.

$$29{,}900{,}000{,}000 = 2.99 \times 10^{10}$$
$$0.000549 = 5.49 \times 10^{-4}$$

Writing Numbers in Scientific Notation

To transform numbers from decimal form to scientific notation, it must be remembered that the laws of exponents form the basis for calculations using powers.

Using the results of the previous chapter, the following whole numbers and decimals can be expressed as powers of 10:

$1 = 10^0$	$0.1 = 1/10 = 10^{-1}$
$10 = 10^1$	$0.01 = 1/100 = 10^{-2}$
$100 = 10^2$	$0.001 = 1/1000 = 10^{-3}$
$1000 = 10^3$	
$10{,}000 = 10^4$	

A number *N* is in scientific notation when it is expressed as the product of a decimal number between 1 and 10 and some integer power of 10.

$$N = a \text{ x } 10^n \quad \text{where } 1 \leq a < 10 \text{ and } n \text{ is an integer.}$$

The steps for converting to scientific notation are as follows:

Step 1: Place the decimal immediately to the right of the left-most non-zero number.

Step 2: Count the number of digits between the old and new decimal point.

Step 3: If the decimal is shifted to the left, the exponent is positive. If the decimal is shifted to the right, the exponent is negative.

Let us examine the logic of this. Consider as an example the number 3750. The number will not be changed if it is multiplied by 1000 and divided by 1000 (the net effect is to multiply it by one). Then,

$$\frac{3750}{1000} \times 1000 = 3.750 \times 1000 = 3.750 \times 10^3$$

There is a division by 10 for each space the decimal point is moved to the left, which is compensated for by multiplying by 10. Similarly, for a number such as .0037, we multiply the number by 10 for each space the decimal point is moved to the right. Thus, the number must be divided by 10 for each space.

$$.0037 = .0\,0\,3\,7 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{3.7}{10^3} = 3.7 \times 10^{-3}$$

Example 1:

Circulating water flows at 440,000 gallons per minute. Express this number in scientific notation.

Solution:

440,000 becomes 4.4×10^{n}

$4\ \underbrace{4}_{5}\ \underbrace{0}_{4}\ \underbrace{0}_{3}\ \underbrace{0}_{2}\ \underbrace{0.}_{1}$

$n = +5$ because the decimal is shifted five places to the left.

$440{,}000 = 4.4 \times 10^{5}$

Example 2:

Express 0.0000247 in scientific notation.

Solution:

$0.\ \underbrace{0}_{1}\ \underbrace{0}_{2}\ \underbrace{0}_{3}\ \underbrace{0}_{4}\ \underbrace{2}_{5}\ 4\ \ 7$

$n = -5$ because the decimal is shifted five places to the right.

$0.0000247 = 2.47 \times 10^{-5}$

Example 3:

Express 34.2 in scientific notation.

Solution:

$3\ \underbrace{4.}_{1}\ 2$

$n = 1$ because the decimal is shifted one place to the left.

$34.2 = 3.42 \times 10^{1}$

<u>Converting Scientific Notation to Integers</u>

Often, numbers in scientific notation need to be put in integer form.

To convert scientific notation to integers:

Step 1: Write decimal number.

Step 2: Move the decimal the number of places specified by the power of ten: to the right if positive, to the left if negative. Add zeros if necessary.

Step 3: Rewrite the number in integer form.

Example:

Convert 4.4×10^3 to integer form.

Solution:

Step 1: 4.4

Step 2: $4 . \underbrace{4}_{1}\ \underbrace{0}_{2}\ \underbrace{0}_{3}$

Step 3: $4.4 \times 10^3 = 4400$

Addition

In order to add two or more numbers using scientific notation, the following three steps must be used.

Step 1: Change all addends to have the same power of ten by moving the decimal point (that is, change all lower powers of ten to the highest power).

Step 2: Add the decimal numbers of the addends and keep the common power of ten.

Step 3: If necessary, rewrite the decimal with a single number to the left of the decimal point.

For example, for $3.5 \times 10^3 + 5 \times 10^2$ you are asked to add 3.5 thousands to 5 hundreds. Converting 3.5 thousands to 35 hundreds ($3.5 \times 10^3 = 35 \times 10^2$) we obtain 35 hundreds + 5 hundreds = 40 hundreds or $3.5 \times 10^3 = 35 \times 10^2 + 5 \times 10^2 = 4 \times 10^3$. The student should do the same problem by converting the 5×10^2 to thousands and then adding.

Example:

Add $(9.24 \times 10^4) + (8.3 \times 10^3)$

Solution:

Step 1: $9.24 \times 10^4 = 9.24 \times 10^4$
$8.3 \times 10^3 = 0.83 \times 10^4$

Step 2: $\quad 9.24 \times 10^4$
$\underline{+0.83 \times 10^4}$

Step 3: $\quad 10.07 \times 10^4 = 1.007 \times 10^5$

Subtraction

In order to subtract two numbers in scientific notation, the steps listed below must be followed.

Step 1: As in addition, change all addends to have the same power of ten.

Step 2: Subtract one digit from the other and keep the power of ten.

Step 3: If necessary, rewrite the decimal with a single number to the left of the decimal point.

Example:

Subtract $(3.27 \times 10^4) - (2 \times 10^3)$

Solution:

Step 1: $3.27 \times 10^4 = 3.27 \times 10^4$
$2.00 \times 10^3 = 0.20 \times 10^4$

Step 2: $\quad 3.27 \times 10^4$
$\underline{-0.20 \times 10^4}$
Step 3: $\quad 3.07 \times 10^4$

Multiplication

When multiplying two or more numbers in scientific notation, the following steps must be used.

Step 1: Multiply the decimal numbers and obtain the product.

Step 2: Multiply the powers of ten together by adding the exponents.

Step 3: Put the product in single-digit scientific notation.

Step 4: If necessary, rewrite decimal with a single number to the left of the decimal point.

Example:

Multiply $(3 \times 10^3)(5 \times 10^{-2})$

Solution:

Step 1: $3 \times 5 = 15$

Step 2: $10^3 \times 10^{-2} = 10^{3 + -2} = 10^1$

Step 3: The product is: 15×10^1

Step 4: $= 1.5 \times 10^2$

Division

Follow the steps listed below when dividing numbers in scientific notation.

Step 1: Divide one decimal into the other.

Step 2: Divide one power of ten into the other by subtracting the exponents.

Step 3: Put product in single-digit scientific notation.

Step 4: If necessary, rewrite decimal with a single number to the left of the decimal point.

Example:

$(1 \times 10^6) \div 5 \times 10^4 =$

Solution:

Step 1: $\frac{1}{5} = 0.2$

Step 2: $\frac{10^6}{10^4} = 10^{(6-4)} = 10^2$

Step 3: 0.2×10^2

Step 4: 2.0×10^1

Summary

Pertinent information concerning scientific notation is summarized below.

Scientific Notation Summary

When changing from integer form to scientific notation:

- If the decimal is shifted left, the exponent is positive.
- If the decimal is shifted right, the exponent is negative.

When adding or subtracting numbers in scientific notation, change both numbers to the same power of ten by moving the decimal point. Add or subtract the decimal numbers, and keep the power of ten. Rewrite if necessary.

To multiply two numbers in scientific notation, multiply decimal numbers and add exponents. Rewrite if necessary.

To divide two numbers in scientific notation, divide decimal numbers and subtract exponents. Rewrite if necessary.

RADICALS

This chapter covers the addition, subtraction, multiplication, and division of radicals.

EO 1.15 CALCULATE the numerical value of numbers in radical form.

Calculator Usage, Special Keys

The exponent key can be used for radicals if the exponent is entered in decimal form.

Exponent key

y^x

Raising a number to an exponent requires the y^x key to be pressed twice. First, the base number is entered and the y^x key is pressed. This enters the base number (y). Next, the exponent number is entered and the y^x key is pressed. This enters the exponent and tells the calculator to complete the calculation. The calculator will display the value.

Square-root key

$\sqrt{x}$

Pressing this key takes the square root of the displayed number.

The Radical

A previous chapter explained how to raise a number to a power. The inverse of this operation is called extracting a root. For any positive integer n, a number x is the nth root of the number a if it satisfies $x^n = a$. For example, since $2^5 = 32$, 2 is the fifth root of 32.

To indicate the nth root of a, the expression $a^{1/n}$ is often used. The symbol $\sqrt{\ }$ is called the radical sign, and the nth root of a can also be shown as $\sqrt[n]{a}$. The letter a is the radicand, and n is the index. The index 2 is generally omitted for square roots.

Example:

$$\sqrt{4} = 2$$

$$\sqrt[3]{27} = 3$$

$$\sqrt[4]{x^4} = x$$

Simplifying Radicals

An expression having radicals is in simplest form when:

- The index cannot be reduced.
- The radicand is simplified.
- No radicals are in the denominator.

There are four rules of radicals that will be useful in simplifying them.

Rule 1: $\left(\sqrt[n]{a}\right)^n = \sqrt[n]{a^n} = a$

Rule 2: $\sqrt[n]{ab} = \sqrt[n]{a}\ \sqrt[n]{b}$

Rule 3: $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\left(\sqrt[n]{a}\right)}{\left(\sqrt[n]{b}\right)}$

Rule 4: $\sqrt[n]{-a} = -\sqrt[n]{a}$, when n is odd.

Examples: $\sqrt{10^2} = 10$

$\left(\sqrt[3]{26}\right)^3 = 26$

$\sqrt{27} = \sqrt{9 \cdot 3} = \sqrt{9}\ \sqrt{3} = 3\sqrt{3}$

$\sqrt[3]{-54} = \sqrt[3]{(-27)(2)} = \left(\sqrt[3]{-27}\right)\left(\sqrt[3]{2}\right) = -3\ \sqrt[3]{2}$

When a radical sign exists in the denominator, it is desirable to remove the radical. This is done by multiplying both the numerator and denominator by the radical and simplifying.

Example: $\dfrac{3}{\sqrt{5}} = \dfrac{3}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{3\sqrt{5}}{5}$

Addition and Subtraction

Addition and subtraction of radicals may be accomplished with radicals showing the same radicand and the same index. Add or subtract similar radicals using the distributive law.

Examples: $3\sqrt{ab} + 2\sqrt{ab} = (3 + 2)\sqrt{ab} = 5\sqrt{ab}$

$7\sqrt{5} - 3\sqrt{5} = (7 - 3)\sqrt{5} = 4\sqrt{5}$

Multiplication

Multiplication of radicals having the same index may be accomplished by applying the rule used in simplification: $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Examples: $\sqrt[3]{3x^4} \cdot \sqrt[3]{9x^2} = \sqrt[3]{27x^6} = 3x^2$

$\sqrt{xy} \cdot \sqrt{3x} = \sqrt{3x^2y} = x\sqrt{3y}$

$$\sqrt[4]{\frac{16}{3}} \cdot \sqrt[4]{\frac{2}{27}} = \sqrt[4]{\frac{32}{81}} = \frac{\sqrt[4]{32}}{\sqrt[4]{81}} = \frac{\sqrt[4]{2 \cdot 16}}{\sqrt[4]{81}} = \frac{2\sqrt[4]{2}}{3}$$

Division

Division of radicals having the same index, but not necessarily the same radicand, may be performed by using the following rule and simplifying.

Examples: $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

$$\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3$$

Dissimilar Radicals

Often, dissimilar radicals may be combined after they are simplified.

Example: $\sqrt[4]{81x^2} + \sqrt{x} - \sqrt[6]{64x^3}$

$= 3\sqrt{x} + \sqrt{x} - 2\sqrt{x}$

$= (3+1-2)\sqrt{x} = 2\sqrt{x}$

Changing Radicals to Exponents

This chapter has covered solving radicals and then converting them into exponential form. It is much easier to convert radicals to exponential form and then perform the indicated operation.

The expression $\sqrt[3]{4}$ can be written with a fractional exponent as $4^{1/3}$. Note that this meets the condition $\left(4^{\frac{1}{3}}\right)^3 = 4$, that is, the cube root of 4 cubed equals 4. This can be expressed in the following algebraic form:

$$a^{1/n} = \sqrt[n]{a}$$

The above definition is expressed in more general terms as follows:

$$a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

Example 1:

Express the following in exponential form.

$$\sqrt[3]{27^2} = 27^{2/3}$$

$$\sqrt{2} = 2^{1/2}$$

Example 2:

Solve the following by first converting to exponential form.

$$\sqrt{27} \cdot \sqrt[3]{27} = 27^{1/2} \cdot 27^{1/3} = 27^{5/6}$$

but $27 = 3^3$

substituting: $27^{5/6} = (3^3)^{5/6} = 3^{5/2}$

Changing Exponents to Radicals

How to convert radicals into exponential form has been explained. Sometimes however, it is necessary or convenient to convert exponents to radicals. Recognizing that an exponent is the equivalent of the n^{th} root is useful to help comprehend an expression.

The expression $5^{1/3}$ can be written as $\sqrt[3]{5}$. It is algebraically expressed as:

$$a^{\frac{1}{m}} = \sqrt[m]{a}$$

The above definition can be more generally described as:

$$\sqrt[n]{a^m} = (a^m)^{\frac{1}{n}} = a^{\frac{m}{n}}$$

and

$$\left(\sqrt[n]{b}\right)^m = \left(b^{\frac{1}{n}}\right)^m = b^{\frac{m}{n}}$$

Examples: $15^{2/3} = \sqrt[3]{15^2}$

$16^{1/2} = \sqrt[2]{16} = 4$

Summary

Pertinent information concerning radicals is summarized below.

Radicals Summary

- $\left(\sqrt[n]{a}\right)^n = \sqrt[n]{a^n} = a$ Used in simplification
- $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$ Used in simplification and multiplication
- $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\left(\sqrt[n]{a}\right)}{\left(\sqrt[n]{b}\right)}$ Used in simplification and division
- $\sqrt[n]{a} = a^{1/n}$ Used to change radicals to exponents and exponents to radicals

Appendix A

TI-30 Keyboard

Review of Introductory Mathematics

Figure A-1 TI-30 Keyboard Layout