# MATHEMATICS
# Module 3
# Geometry

# TABLE OF CONTENTS

# LIST OF FIGURES

# REFERENCES


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# TERMINAL OBJECTIVE

1.0 Given a calculator and the correct formula, **APPLY** the laws of geometry to solve mathematical problems.

# ENABLING OBJECTIVES

1.1 **IDENTIFY** a given angle as either:
- a. Straight
- b. Acute
- c. Right
- d. Obtuse

1.2 **STATE** the definitions of complimentary and supplementary angles.

1.3 **STATE** the definition of the following types of triangles:
- a. Equilateral
- b. Isosceles
- c. Acute
- d. Obtuse
- e. Scalene

1.4 Given the formula, **CALCULATE** the area and the perimeter of each of the following basic geometric shapes:
- a. Triangle
- b. Parallelogram
- c. Circle

1.5 Given the formula, **CALCULATE** the volume and surface areas of the following solid figures:
- a. Rectangular solid
- b. Cube
- c. Sphere
- d. Right circular cone
- e. Right circular cylinder

# BASIC CONCEPTS OF GEOMETRY

*This chapter covers the basic language and terminology of plane geometry.*

**EO 1.1 IDENTIFY a given angle as either:**
**a. Straight**
**b. Acute**
**c. Right**
**d. Obtuse**

**EO 1.2 STATE the definitions of complimentary and supplementary angles.**

Geometry is one of the oldest branches of mathematics. Applications of geometric constructions were made centuries before the mathematical principles on which the constructions were based were recorded. Geometry is a mathematical study of points, lines, planes, closed flat shapes, and solids. Using any one of these alone, or in combination with others, it is possible to describe, design, and construct every visible object.

The purpose of this section is to provide a foundation of geometric principles and constructions on which many practical problems depend for solution.

## Terms

There are a number of terms used in geometry.

1. A *plane* is a flat surface.
2. *Space* is the set of all points.
3. *Surface* is the boundary of a solid.
4. *Solid* is a three-dimensional geometric figure.
5. *Plane geometry* is the geometry of planar figures (two dimensions). Examples are: angles, circles, triangles, and parallelograms.
6. *Solid geometry* is the geometry of three-dimensional figures. Examples are: cubes, cylinders, and spheres.

## Lines

A *line* is the path formed by a moving point. A *length of a straight line* is the shortest distance between two nonadjacent points and is made up of collinear points. A *line segment* is a portion of a line. A *ray* is an infinite set of collinear points extending from one end point to infinity. A set of points is noncollinear if the points are not contained in a line.

Two or more straight lines are *parallel* when they are coplanar (contained in the same plane) and do not intersect; that is, when they are an equal distance apart at every point.

## Important Facts

The following facts are used frequently in plane geometry. These facts will help you solve problems in this section.

1. The shortest distance between two points is the length of the straight line segment joining them.
2. A straight line segment can be extended indefinitely in both directions.
3. Only one straight line segment can be drawn between two points.
4. A geometric figure can be moved in the plane without any effect on its size or shape.
5. Two straight lines in the same plane are either parallel or they intersect.
6. Two lines parallel to a third line are parallel to each other.

## Angles

An *angle* is the union of two nonparallel rays originating from the same point; this point is known as the vertex. The rays are known as sides of the angle, as shown in Figure 1.

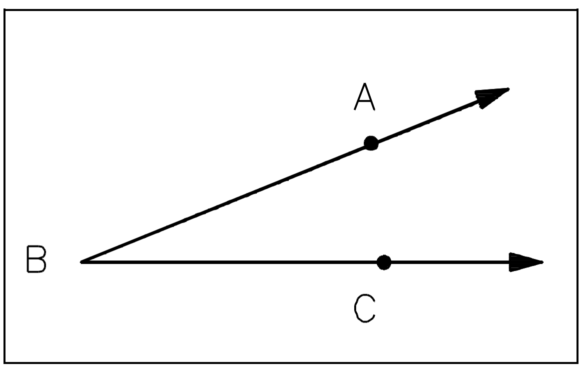


Figure 1 Angle

If ray *AB* is on top of ray *BC*, then the angle *ABC* is a *zero angle*. One complete revolution of a ray gives an angle of 360°.

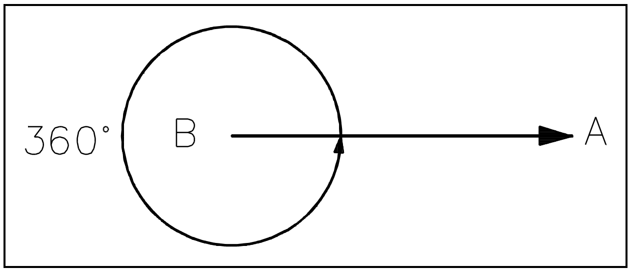


Figure 2 - 360º Angle

Depending on the rotation of a ray, an angle can be classified as right, straight, acute, obtuse, or reflex. These angles are defined as follows:

Right Angle - angle with a ray separated by 90°.

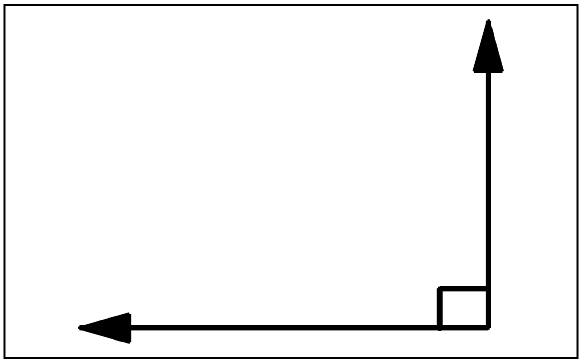

Figure 3 Right Angle

Straight Angle - angle with a ray separated by 180° to form a straight line.

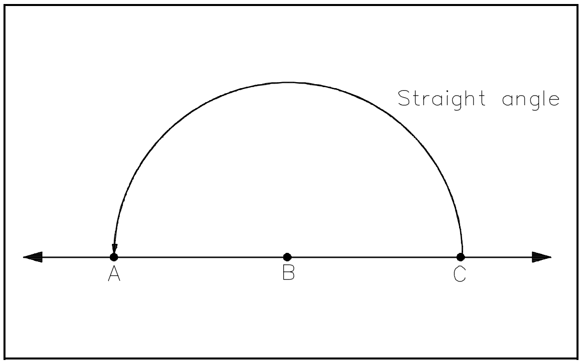


Figure 4 Straight Angle

Acute Angle - angle with a ray separated by less than 90°.

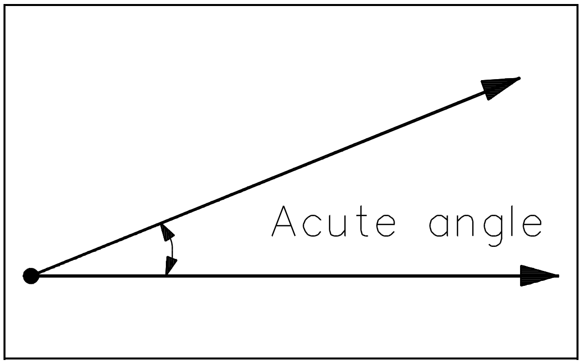


Figure 5 Acute Angle

Obtuse Angle - angle with a ray rotated greater than 90° but less than 180°.

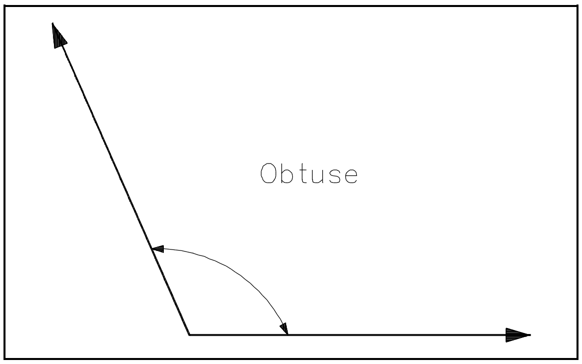


Figure 6 Obtuse Angle

Reflex Angle - angle with a ray rotated greater than 180°.

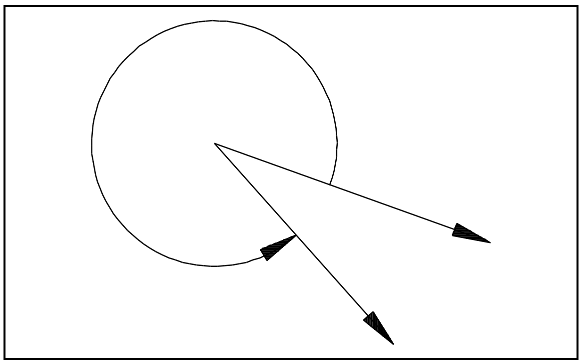

Figure 7 Reflex Angle

If angles are next to each other, they are called *adjacent angles.* If the sum of two angles equals 90°, they are called *complimentary angles*. For example, 27° and 63° are complimentary angles. If the sum of two angles equals 180°, they are called *supplementary angles*. For example, 73° and 107° are supplementary angles.

## Summary

The important information in this chapter is summarized below.

### Lines and Angles Summary

- Straight lines are parallel when they are in the same plane and do not intersect.
- A straight angle is 180°.
- An acute angle is less than 90°.
- A right angle is 90°.
- An obtuse angle is greater than 90° but less than 180°.
- If the sum of two angles equals 90°, they are complimentary angles.
- If the sum of two angles equals 180°, they are supplementary angles.

# SHAPES AND FIGURES OF PLANE GEOMETRY

*This chapter covers the calculation of the perimeter and area of selected plane figures.*

**EO 1.3 STATE the definition of the following types of triangles:**
**a. Equilateral**
**b. Isosceles**
**c. Acute**
**d. Obtuse**
**e. Scalene**

**EO 1.4 Given the formula, CALCULATE the area and the perimeter of each of the following basic geometric shapes:**
**a. Triangle**
**b. Parallelogram**
**c. Circle**

The terms and properties of lines, angles, and circles may be applied in the layout, design, development, and construction of closed flat shapes. A new term, plane, must be understood in order to accurately visualize a closed, flat shape. A plane refers to a flat surface on which lies a straight line connecting any two points.

A plane figure is one which can be drawn on a plane surface. There are many types of plane figures encountered in practical problems. Fundamental to most design and construction are three flat shapes: the triangle, the rectangle, and the circle.

## Triangles

A *triangle* is a figure formed by using straight line segments to connect three points that are not in a straight line. The straight line segments are called sides of the triangle.

Examples of a number of types of triangles are shown in Figure 8. An *equilateral triangle* is one in which all three sides and all three angles are equal. Triangle *ABC* in Figure 8 is an example of an equilateral triangle. An *isosceles triangle* has two equal sides and two equal angles (triangle *DEF*). A *right triangle* has one of its angles equal to 90° and is the most important triangle for our studies (triangle *GHI*). An *acute triangle* has each of its angles less than 90° (triangle *JKL*). Triangle *MNP* is called a *scalene triangle* because each side is a different length. Triangle *QRS* is considered an *obtuse triangle* since it has one angle greater than 90°. A triangle may have more than one of these attributes. The sum of the interior angles in a triangle is always 180°.

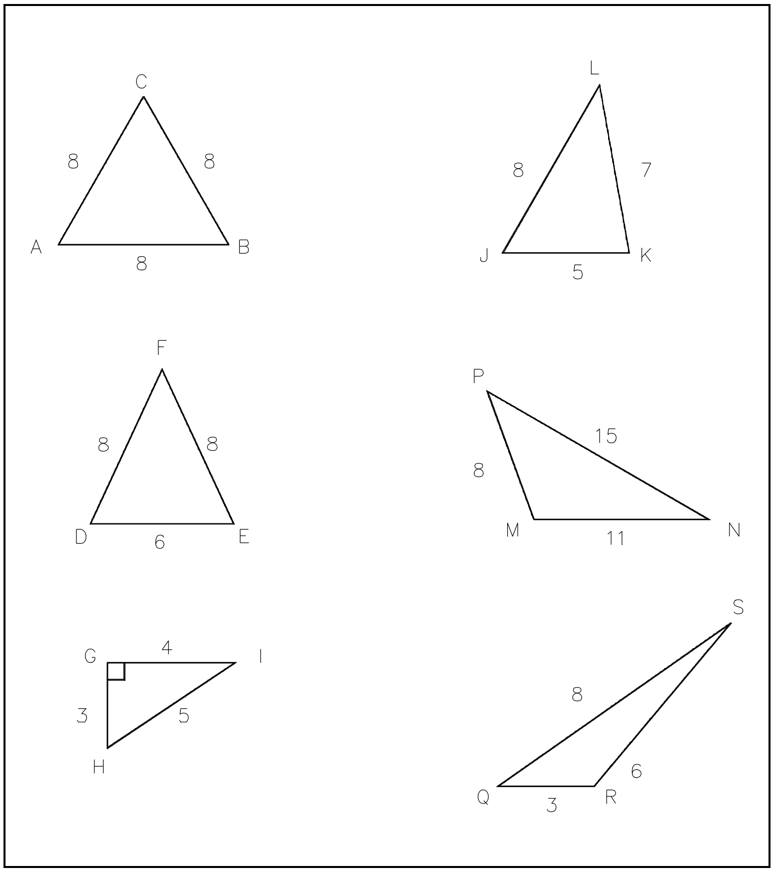


Figure 8 Types of Triangles

## Area and Perimeter of Triangles

The area of a triangle is calculated using the formula:

$$A = (1/2)(base) \bullet (height) \quad (3\text{-}1)$$

or

$$A = (1/2)bh$$

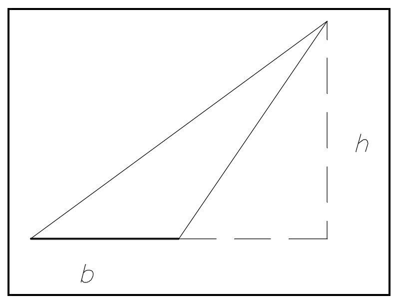


Figure 9 Area of a Triangle

The perimeter of a triangle is calculated using the formula:

$$P = side_1 + side_2 + side_3. \quad (3\text{-}2)$$

The area of a traingle is always expressed in square units, and the perimeter of a triangle is always expressed in the original units.

Example:

Calculate the area and perimeter of a right triangle with a 9" base and sides measuring 12" and 15". Be sure to include the units in your answer.

Solution:

$A = 1/2\ bh$
$A = .5(9)(12)$
$A = .5(108)$
$A = 54$ square inches

$P = s_1 + s_2 + b$
$P = 9 + 12 + 15$
$P = 36$ inches

## Quadrilaterals

A *quadrilateral* is any four-sided geometric figure.

A *parallelogram* is a four-sided quadrilateral with both pairs of opposite sides parallel, as shown in Figure 10.

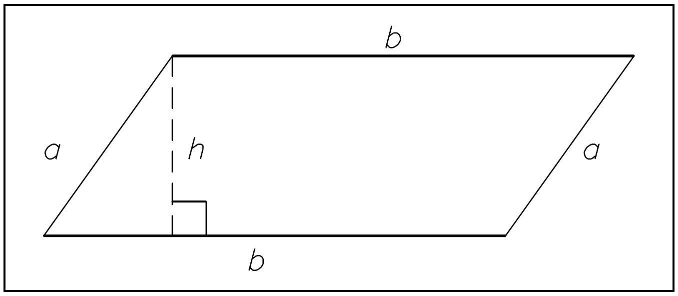


Figure 10 Parallelogram

The area of the parallelogram is calculated using the following formula:

$$A = (base) \bullet (height) = bh \quad (3\text{-}3)$$

The perimeter of a parallelogram is calculated using the following formula:

$$P = 2a + 2b \quad (3\text{-}4)$$

The area of a parallelogram is always expressed in square units, and the perimeter of a parallelogram is always expressed in the original units.

Example:

Calculate the area and perimeter of a parallelogram with base *(b)* = 4´, height *(h)* = 3´, *a* = 5´ and *b* = 4´. Be sure to include units in your answer.

Solution:

| | |
|---|---|
| $A = bh$ | $P = 2a + 2b$ |
| $A = (4)(3)$ | $P = 2(5) + 2(4)$ |
| $A = 12$ square feet | $P = 10 + 8$ |
| | $P = 18$ feet |

A *rectangle* is a parallelogram with four right angles, as shown in Figure 11.

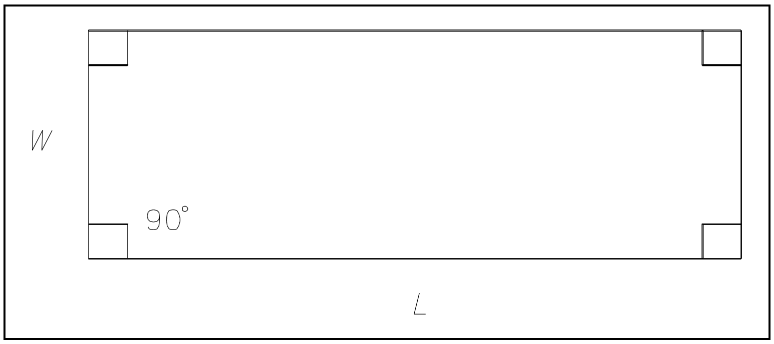


Figure 11 Rectangle

The area of a rectangle is calculated using the following formula:

$$A = (length) \bullet (width) = lw \quad (3\text{-}5)$$

The perimeter of a rectangle is calculated using the following formula:

$$P = 2(length) + 2(width) = 2l + 2w \quad (3\text{-}6)$$

The area of a rectangle is always expressed in square units, and the perimeter of a rectangle is always expressed in the original units.

Example:

Calculate the area and perimeter of a rectangle with $w$ = 5´ and $l$ = 6´. Be sure to include units in your answer.

Solution:

$A = lw$ $P = 2l + 2w$
$A = (5)(6)$ $P = 2(5) + 2(6)$
$A$ = 30 square feet $P = 10 + 12$
$P$ = 22 feet

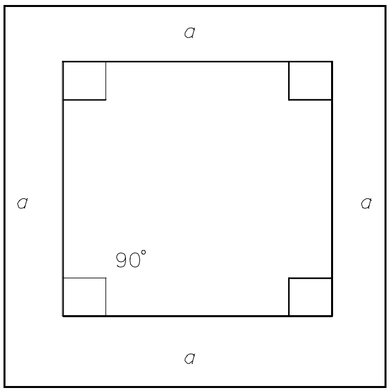


Figure 12 Square

A *square* is a rectangle having four equal sides, as shown in Figure 12.

The area of a square is calculated using the following formula:

$$A = a^2 \tag{3-7}$$

The perimeter of a square is calculated using the following formula:

$$A = 4a \tag{3-8}$$

The area of a square is always expressed in square units, and the perimeter of a square is always expressed in the original units.

Example:

Calculate the area and perimeter of a square with $a$ = 5´. Be sure to include units in your answer.

Solution:

$A = a^2$ $P = 4a$
$A = (5)(5)$ $P = 4(5)$
$A$ = 25 square feet $P$ = 20 feet

## Circles

A *circle* is a plane curve which is equidistant from the center, as shown in Figure 13. The length of the perimeter of a circle is called the *circumference*. The *radius (r)* of a circle is a line segment that joins the center of a circle with any point on its circumference. The *diameter* (*D*) of a circle is a line segment connecting two points of the circle through the center. The area of a circle is calculated using the following formula:

$$A = \pi r^2 \tag{3-9}$$

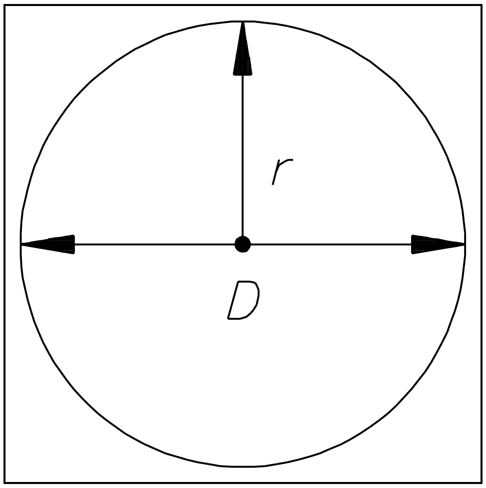


Figure 13 Circle

The circumference of a circle is calculated using the following formula:

$$C = 2\pi r \tag{3-10}$$

or

$$C = \pi D$$

Pi ($\pi$) is a theoretical number, approximately 22/7 or 3.141592654, representing the ratio of the circumference to the diameter of a circle. The scientific calculator makes this easy by designating a key for determining $\pi$.

The area of a circle is always expressed in square units, and the perimeter of a circle is always expressed in the original units.

Example:

Calculate the area and circumference of a circle with a 3" radius. Be sure to include units in your answer.

Solution:

| | |
|---|---|
| $A = \pi r^2$ | $C = 2\pi r$ |
| $A = \pi(3)(3)$ | $C = (2)\pi(3)$ |
| $A = \pi(9)$ | $C = \pi(6)$ |
| $A$ = 28.3 square inches | $C$ = 18.9 inches |

## Summary

The important information in this chapter is summarized below.

### Shapes and Figures of Plane Geometry Summary

| | | |
|---|---|---|
| • Equilateral Triangle | - | all sides equal |
| • Isosceles Triangle | - | 2 equal sides and 2 equal angles |
| • Right Triangle | - | 1 angle equal to 90° |
| • Acute Triangle | - | each angle less than 90° |
| • Obtuse Triangle | - | 1 angle greater than 90° |
| • Scalene Triangle | - | each side a different length |
| • Area of a triangle | - | $A = (1/2)(base) \bullet (height)$ |
| • Perimeter of a triangle | - | $P = side_1 + side_2 + side_3$ |
| • Area of a parallelogram | - | $A = (base) \bullet (height)$ |
| • Perimeter of a parallelogram | - | $P = 2a + 2b$ where $a$ and $b$ are length of sides |
| • Area of a rectangle | - | $A = (length) \bullet (width)$ |
| • Perimeter of a rectangle | - | $P = 2(length) + 2(width)$ |
| • Area of a square | - | $A = edge^2$ |
| • Perimeter of a square | - | $P = 4 \times edge$ |
| • Area of a circle | - | $A = \pi r^2$ |
| • Circumference of a circle | - | $C = 2\pi r$ |

# SOLID GEOMETRIC FIGURES

*This chapter covers the calculation of the surface area and volume of selected solid figures.*

**EO 1.5 Given the formula, CALCULATE the volume and surface areas of the following solid figures:**

**a. Rectangular solid**
**b. Cube**
**c. Sphere**
**d. Right circular cone**
**e. Right circular cylinder**

The three flat shapes of the triangle, rectangle, and circle may become solids by adding the third dimension of depth. The triangle becomes a cone; the rectangle, a rectangular solid; and the circle, a cylinder.

## Rectangular Solids

A *rectangular solid* is a six-sided solid figure with faces that are rectangles, as shown in Figure 14.

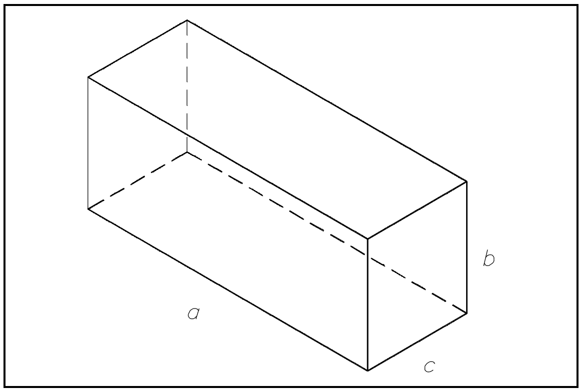


Figure 14 Rectangular Solid

The volume of a rectangular solid is calculated using the following formula:

$$V = abc \quad (3\text{-}11)$$

The surface area of a rectangular solid is calculated using the following formula:

$$SA = 2(ab + ac + bc) \quad (3\text{-}12)$$

The surface area of a rectangular solid is expressed in square units, and the volume of a rectangular solid is expressed in cubic units.

Example:

Calculate the volume and surface area of a rectangular solid with $a$ = 3", $b$ = 4", and $c$ = 5". Be sure to include units in your answer.

Solution:

| | |
|---|---|
| $V = (a)(b)(c)$ | $SA = 2(ab + ac + bc)$ |
| $V = (3)(4)(5)$ | $SA = 2[(3)(4) + (3)(5) + (4)(5)]$ |
| $V = (12)(5)$ | $SA = 2[12 + 15 + 20]$ |
| $V$ = 60 cubic inches | $SA = 2[47]$ |
| | $SA$ = 94 square inches |

## Cube

A *cube* is a six-sided solid figure whose faces are congruent squares, as shown in Figure 15.

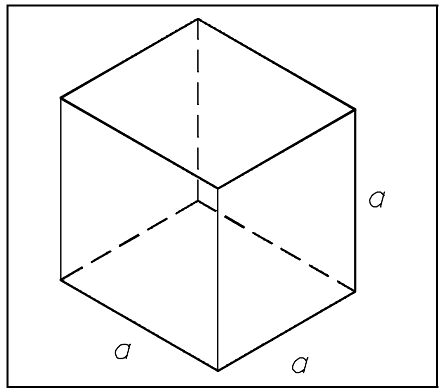


Figure 15 Cube

The volume of a cube is calculated using the following formula:

$$V = a^3 \tag{3-13}$$

The surface area of a cube is calculated using the following formula:

$$SA = 6a^2 \tag{3-14}$$

The surface area of a cube is expressed in square units, and the volume of a cube is expressed in cubic units.

Example:

Calculate the volume and surface area of a cube with $a$ = 3". Be sure to include units in your answer.

Solution:

| | |
|---|---|
| $V = a^3$ | $SA = 6a^2$ |
| $V = (3)(3)(3)$ | $SA = 6(3)(3)$ |
| $V$ = 27 cubic inches | $SA = 6(9)$ |
| | $SA$ = 54 square inches |

## Sphere

A *sphere* is a solid, all points of which are equidistant from a fixed point, the center, as shown in Figure 16.

The volume of a sphere is calculated using the following formula:

$$V = 4/3\pi r^3 \quad (3\text{-}15)$$

The surface area of a sphere is calculated using the following formula:

$$SA = 4\pi r^2 \quad (3\text{-}16)$$

The surface area of a sphere is expressed in square units, and the volume of a sphere is expressed in cubic units.

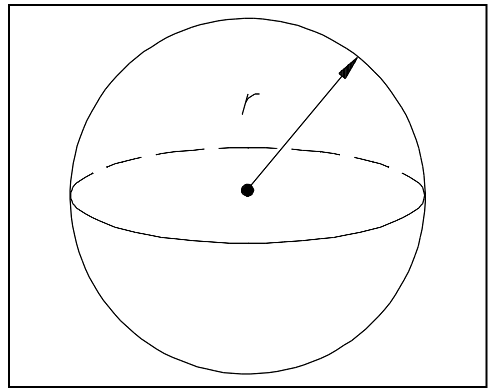


Figure 16 Sphere

Example:

Calculate the volume and surface area of a sphere with $r$ = 4". Be sure to include units in your answer.

Solution:

$V = 4/3\pi r^3$
$V = 4/3\pi(4)(4)(4)$
$V = 4.2(64)$
$V$ = 268.8 cubic inches

$SA = 4\pi r^2$
$SA = 4\pi(4)(4)$
$SA = 12.6(16)$
$SA$ = 201.6 square inches

## Right Circular Cone

A *right circular cone* is a cone whose axis is a line segment joining the vertex to the midpoint of the circular base, as shown in Figure 17.

The volume of a right circular cone is calculated using the following formula:

$$V = 1/3\pi r^2 h \quad (3\text{-}17)$$

The surface area of a right circular cone is calculated using the following formula:

$$SA = \pi r^2 + \pi r l \quad (3\text{-}18)$$

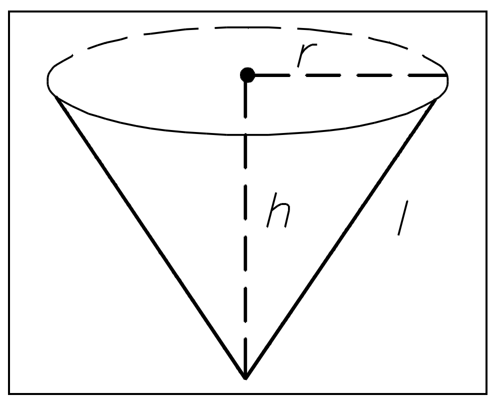


Figure 17 Right Circular Cone

The surface area of a right circular cone is expressed in square units, and the volume of a right circular cone is expressed in cubic units.

Example:

Calculate the volume and surface area of a right circular cone with $r$ = 3", $h$ = 4", and $l$ = 5". Be sure to include the units in your answer.

Solution:

$V = 1/3\pi r^2 h$
$V = 1/3\pi(3)(3)(4)$
$V = 1.05(36)$
$V$ = 37.8 cubic inches

$SA = \pi r^2 + \pi rl$
$SA = \pi(3)(3) + \pi(3)(5)$
$SA = \pi(9) + \pi(15)$
$SA = 28.3 + 47.1$
$SA$ = 528/7 = 75-3/7 square inches

## Right Circular Cylinder

A *right circular cylinder* is a cylinder whose base is perpendicular to its sides. Facility equipment, such as the reactor vessel, oil storage tanks, and water storage tanks, is often of this type.

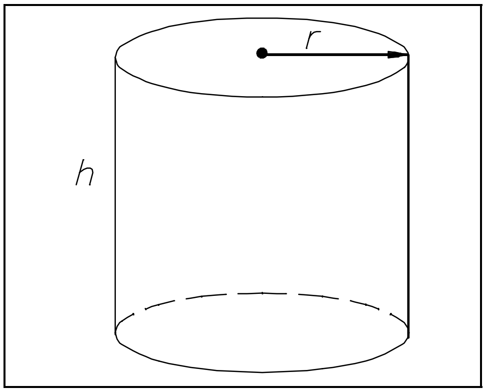

Figure 18 Right Circular Cylinder

The volume of a right circular cylinder is calculated using the following formula:

$$V = \pi r^2 h \quad (3\text{-}19)$$

The surface area of a right circular cylinder is calculated using the following formula:

$$SA = 2\pi rh + 2\pi r^2 \quad (3\text{-}20)$$

The surface area of a right circular cylinder is expressed in square units, and the volume of a right circular cylinder is expressed in cubic units.

Example:

Calculate the volume and surface area of a right circular cylinder with $r$ = 3" and $h$ = 4". Be sure to include units in your answer.

Solution:

$V = \pi r^2 h$
$V = \pi(3)(3)(4)$
$V = \pi(36)$
$V$ = 113.1 cubic inches

$SA = 2\pi rh + 2\pi r^2$
$SA = 2\pi(3)(4) + 2\pi(3)(3)$
$SA = 2\pi(12) + 2\pi(9)$
$SA$ = 132 square inches

## Summary

The important information in this chapter is summarized below.

### Solid Geometric Shapes Summary

- Volume of a rectangular solid: $abc$

  Surface area of a rectangular solid: $2(ab + ac + bc)$
- Volume of a cube: $a^3$
- Surface area of a cube: $6a^2$
- Volume of a sphere: $4/3\pi r^3$
- Surface area of a sphere: $4\pi r^2$
- Volume of a right circular cone: $1/3\pi r^2 h$
- Surface area of a right circular cone: $\pi r^2 + \pi rl$
- Volume of a right circular cylinder: $\pi r^2 h$
- Surface area of right circular cylinder: $2\pi rh + 2\pi r^2$