DOE Fundamentals

ELECTRICAL SCIENCE

Module 3

DC Circuits

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REFERENCES

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OBJECTIVES

TERMINAL OBJECTIVE

1.0 Using the rules associated with inductors and capacitors, **DESCRIBE** the characteristics of these elements when they are placed in a DC circuit.

ENABLING OBJECTIVES

1.1 **DESCRIBE** how current flow, magnetic field, and stored energy in an inductor relate to one another.

1.2 **DESCRIBE** how an inductor opposes a change in current flow.

1.3 Given a circuit containing inductors, **CALCULATE** total inductance for series and parallel circuits.

1.4 Given an inductive resistive circuit, **CALCULATE** the time constant for the circuit.

1.5 **DESCRIBE** the construction of a capacitor.

1.6 **DESCRIBE** how a capacitor stores energy.

1.7 **DESCRIBE** how a capacitor opposes a change in voltage.

1.8 Given a circuit containing capacitors, **CALCULATE** total capacitance for series and parallel circuits.

1.9 Given a circuit containing capacitors and resistors, **CALCULATE** the time constant of the circuit.

INDUCTANCE

Experiments investigating the unique behavioral characteristics of inductance led to the invention of the transformer.

EO 1.1 **DESCRIBE** how current flow, magnetic field, and stored energy in an inductor relate to one another.

EO 1.2 **DESCRIBE** how an inductor opposes a change in current flow.

EO 1.3 Given a circuit containing inductors, **CALCULATE** total inductance for series and parallel circuits.

EO 1.4 Given an inductive resistive circuit, **CALCULATE** the time constant for the circuit.

Inductors

An inductor is a circuit element that will store electrical energy in the form of a magnetic field. It is usually a coil of wire wrapped around a core of permeable material. The magnetic field is generated when current is flowing through the wire. If two circuits are arranged as in Figure 1, a magnetic field is generated around Wire A, but there is no electromotive force (EMF) induced into Wire B because there is no relative motion between the magnetic field and Wire B.

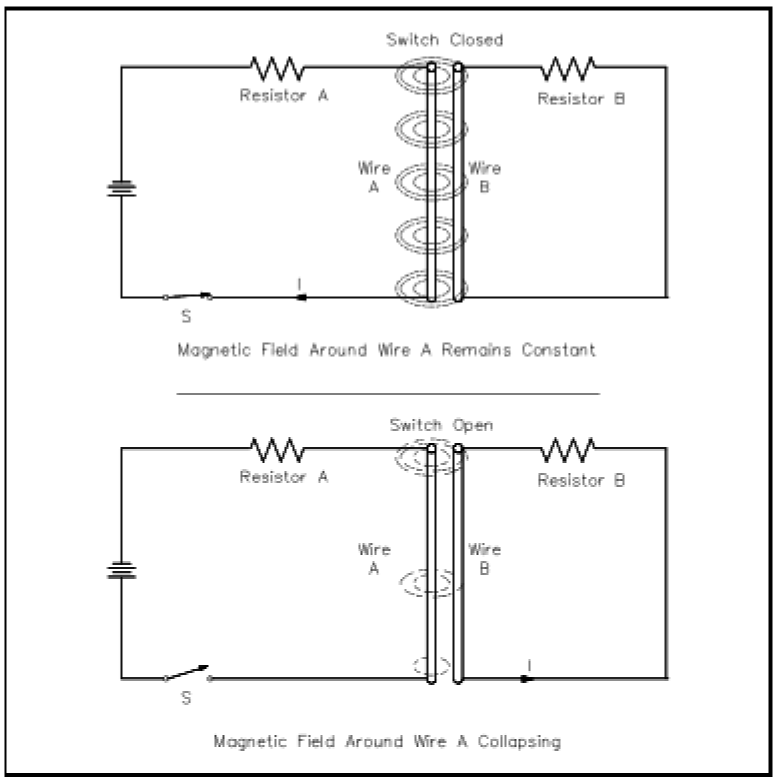

Figure 1 Induced EMF

If we now open the switch, the current stops flowing in Wire A, and the magnetic field collapses. As the field collapses, it moves relative to Wire B. When this occurs, an EMF is induced in Wire B.

This is an example of Faraday's Law, which states that a voltage is induced in a conductor when that conductor is moved through a magnetic field, or when the magnetic field moves past the conductor. When the EMF is induced in Wire B, a current will flow whose magnetic field opposes the change in the magnetic field that produced it.

For this reason, an induced EMF is sometimes called counter EMF or CEMF. This is an example of Lenz's Law, which states that the induced EMF opposes the EMF that caused it.

The three requirements for inducing an EMF are:

1. a conductor,
2. a magnetic field, and
3. relative motion between the two.

The faster the conductor moves, or the faster the magnetic field collapses or expands, the greater the induced EMF. The induction can also be increased by coiling the wire in either Circuit A or Circuit B, or both, as shown in Figure 2.

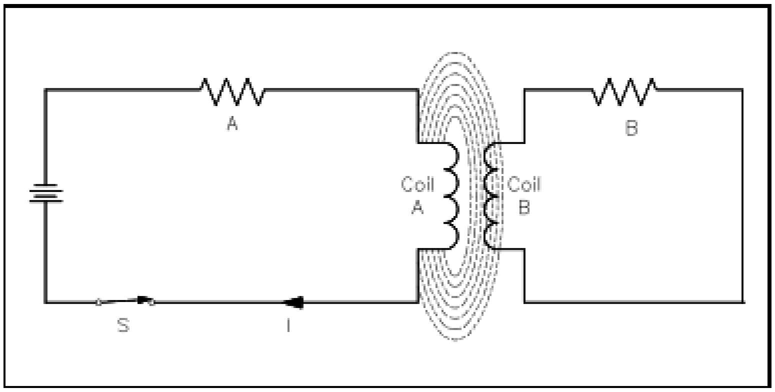

Figure 2 Induced EMF in Coils

Self-induced EMF is another phenomenon of induction. The circuit shown in Figure 3 contains a coil of wire called an inductor (L). As current flows through the circuit, a large magnetic field is set up around the coil. Since the current is not changing, there is no EMF produced. If we open the switch, the field around the inductor collapses. This collapsing magnetic field produces a voltage in the coil. This is called self-induced EMF.

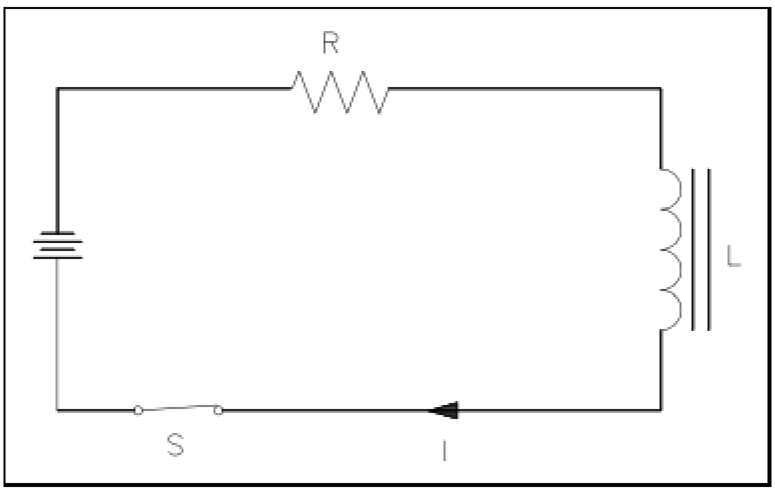

Figure 3 Self-Induced EMF

The polarity of self-induced EMF is given to us by Lenz's Law. The polarity is in the direction that opposes the change in the magnetic field that induced the EMF. The result is that the current caused by the induced EMF tends to maintain the same current that existed in the circuit before the switch was opened. It is commonly said that an inductor tends to oppose a change in current flow.

The induced EMF, or counter EMF, is proportional to the time rate of change of the current. The proportionality constant is called the "inductance" (L). Inductance is a measure of an inductor's ability to induce CEMF. It is measured in henries (H). An inductor has an inductance of one henry if one amp per second change in current produces one volt of CEMF, as shown in Equation (3-1).

$$CEMF = -L\frac{\Delta I}{\Delta t} \tag{3-1}$$

where

CEMF = induced voltage (volts)

L = inductance (henries)

$\Delta I / \Delta t$ = time rate of change of current (amp/sec)

The minus sign shows that the CEMF is opposite in polarity to the applied voltage.

Example: A 4-henry inductor is in series with a variable resistor. The resistance is increased so that the current drops from 6 amps to 2 amps in 2 seconds. What is the CEMF induced?

$$CEMF = -L\frac{\Delta I}{\Delta t}$$

$$= -4\frac{2A - 6A}{2}$$

$$= -4(-2)$$

$$CMEF = +8 \text{ volts}$$

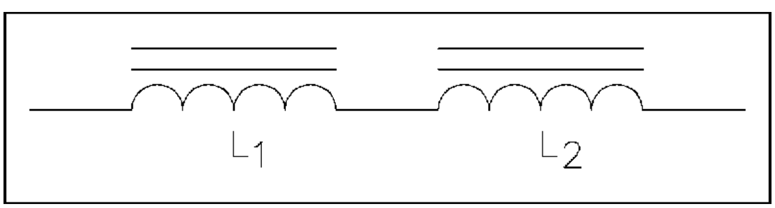

Figure 4 Inductors in Series

Inductors in series are combined like resistors in series. Equivalent inductance (L_{eq}) of two inductors in series (Figure 4) is given by Equation (3-2).

$$L_{eq} = L_1 + L_2 + \dots L_n \tag{3-2}$$

Inductors in parallel are combined like resistors in parallel as given by Equation (3-3).

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots \frac{1}{L_N} \qquad (3\text{-}3)$$

When only two inductors are in parallel, as shown in Figure 5, Equation (3-3) may be simplified as given in Equation (3-4). As shown in Equation (3-4), this is valid when there are only two inductors in parallel.

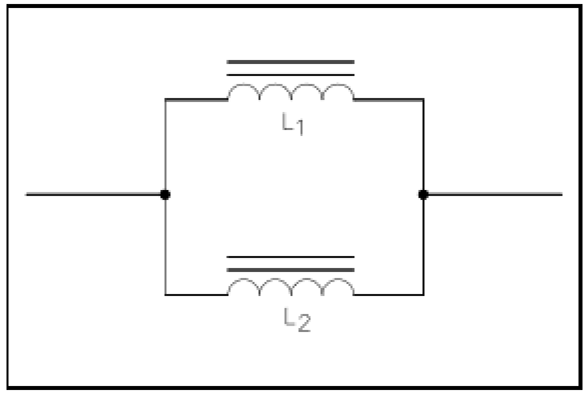

Figure 5 Inductors in Parallel

$$\frac{1}{L_{eq}} = \frac{L_1 L_2}{L_1 + L_2} \qquad (3\text{-}4)$$

Inductors will store energy in the form of a magnetic field. Circuits containing inductors will behave differently from a simple resistance circuit. In circuits with elements that store energy, it is common for current and voltage to exhibit exponential increase and decay (Figure 6).

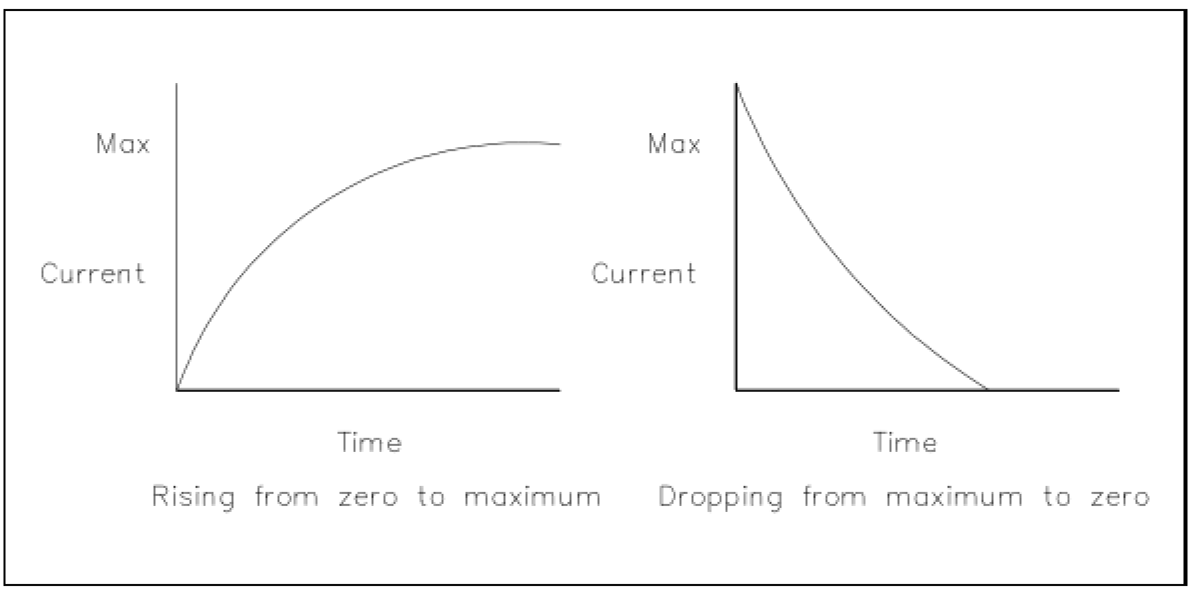

Figure 6 DC Current through an Inductor

The relationship between values of current reached and the time it takes to reach them is called a time constant. The time constant for an inductor is defined as the time required for the current either to increase to 63.2 percent of its maximum value or to decrease by 63.2 percent of its maximum value (Figure 7).

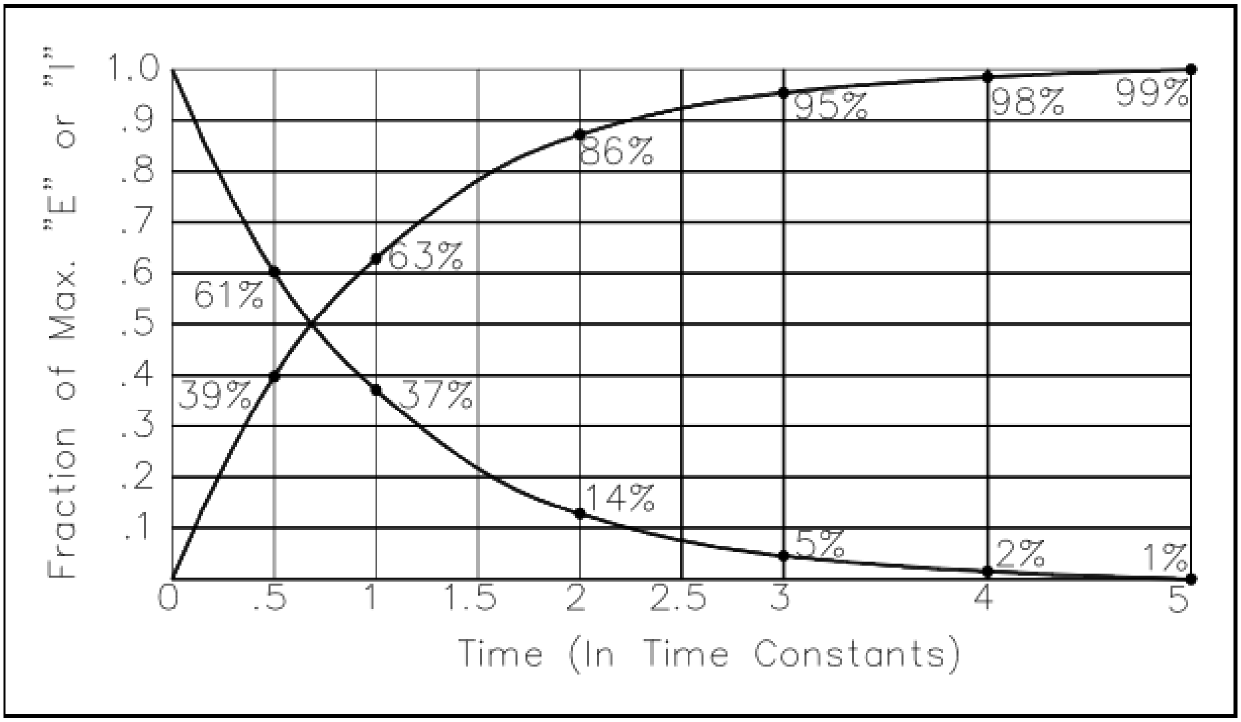

Figure 7 Time Constant

The value of the time constant is directly proportional to the inductance and inversely proportional to the resistance. If these two values are known, the time constant can be found using Equation (3-5).

$$T_L = \frac{L}{R} \tag{3-5}$$

where

T_L	=	time constant (seconds)
L	=	inductance (henries)
R	=	resistance (ohms)

The voltage drop across an inductor is directly proportional to the product of the inductance and the time rate of change of current through the inductor, as shown in Equation (3-6).

$$V_L = L \frac{\Delta I}{\Delta t} \tag{3-6}$$

where

V_L	=	voltage drop across the inductor (volts)
L	=	inductance (henries)
$\Delta I / \Delta t$	=	time rate of change of current (amp/sec)

After five time constants, circuit parameters normally reach their final value. Circuits that contain both inductors and resistors are called RL circuits. The following example will illustrate how an RL circuit reacts to changes in the circuit (Figure 8).

1. Initially, the switch is in Position 1, and no current flows through the inductor.
2. When we move the switch to Position 2, the battery attempts to force a current of 10v/100n = 0.1A through the inductor. But as current begins to flow, the inductor generates a magnetic field. As the field increases, a counter EMF is induced that opposes the battery voltage. As a steady state is reached, the counter EMF goes to zero exponentially.
3. When the switch is returned to Position 1, the magnetic field collapses, inducing an EMF that tends to maintain current flow in the same direction through the inductor. Its polarity will be opposite to that induced when the switch was placed in Position 2.

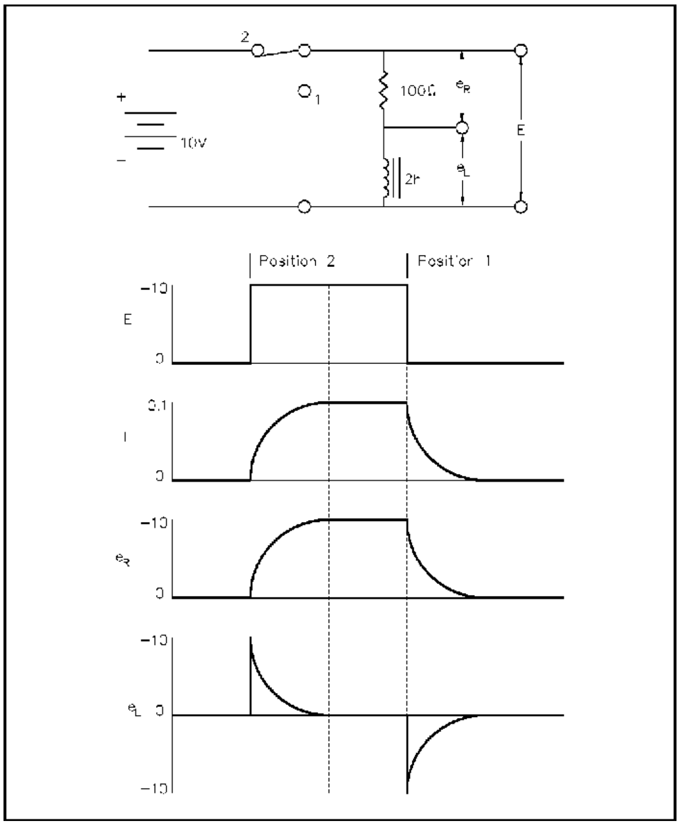

Figure 8 Voltage Applied to an Inductor

The example that follows shows how a circuit with an inductor in parallel with a resistor reacts to changes in the circuit. Inductors have some small resistance, and this is shown schematically as a 1Ω resistor (Figure 9).

1. While the switch is closed, a current of 20 v/l Ω = 20 amps flows through the inductor. This causes a very large magnetic field around the inductor.
2. When we open the switch, there is no longer a current through the inductor. As the magnetic field begins to collapse, a voltage is induced in the inductor. The change in applied voltage is instantaneous; the counter EMF is of exactly the right magnitude to prevent the current from changing initially. In order to maintain the current at 20 amps flowing through the inductor, the self-induced voltage in the inductor must be enough to push 20 amps through the 101Ω of resistance. The CEMF = (101) (20) = 2020 volts.
3. With the switch open, the circuit looks like a series RL circuit without a battery. The CEMF induced falls off, as does the current, with a time constant T_L of:

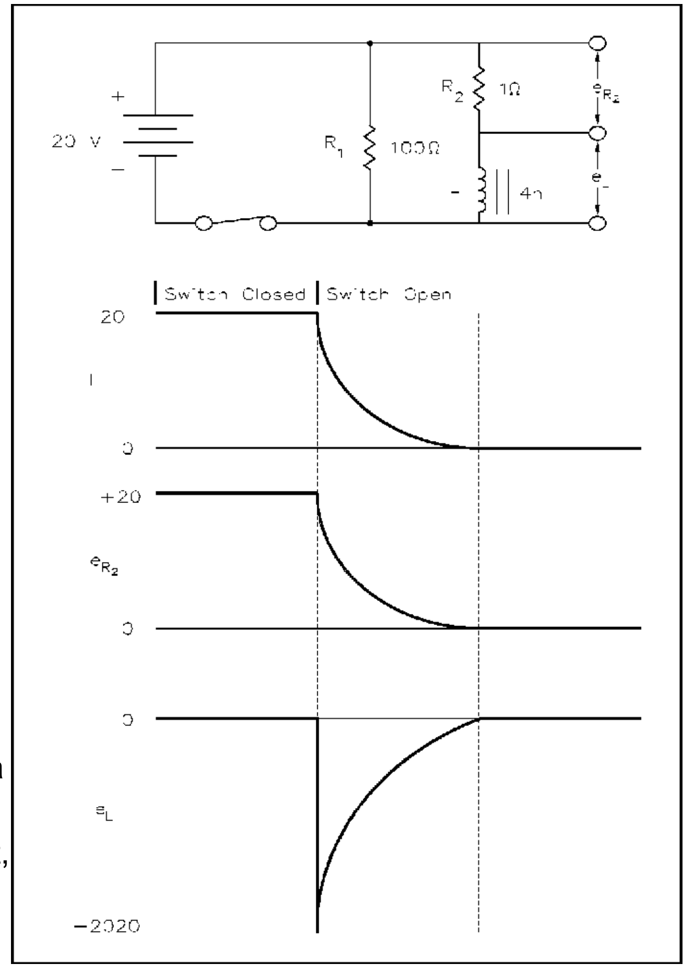

Figure 9 Inductor and Resistor in Parallel

$$T_L = \frac{L}{R}$$

$$T_L = \frac{4H}{101\Omega} = 0.039\text{sec}$$

Summary

The important information on inductors is summarized below.

Inductance Summary

- When an inductor has a DC current flowing through it, the inductor will store energy in the form of a magnetic field.
- An inductor will oppose a change in current flow by the CEMF induced when the field collapses or expands.
- Inductors in series are combined like resistors in series.
- Inductors in parallel are combined like resistors in parallel.
- The time constant for an inductor is defined as the required time for the current either to increase to 63.2 percent of its maximum value or to decrease by 63.2 percent of its maximum value.

CAPACITANCE

Because of the effect of capacitance, an electrical circuit can store energy, even after being de-energized.

EO 1.5 **DESCRIBE** the construction of a capacitor.

EO 1.6 **DESCRIBE** how a capacitor stores energy.

EO 1.7 **DESCRIBE** how a capacitor opposes a change in voltage.

EO 1.8 Given a circuit containing capacitors, **CALCULATE** total capacitance for series and parallel circuits.

EO 1.9 Given a circuit containing capacitors and resistors, **CALCULATE** the time constant of the circuit.

Capacitor

Electrical devices that are constructed of two metal plates separated by an insulating material, called a *dielectric,* are known as capacitors (Figure 10a). Schematic symbols shown in Figures 10b and 10c apply to all capacitors.

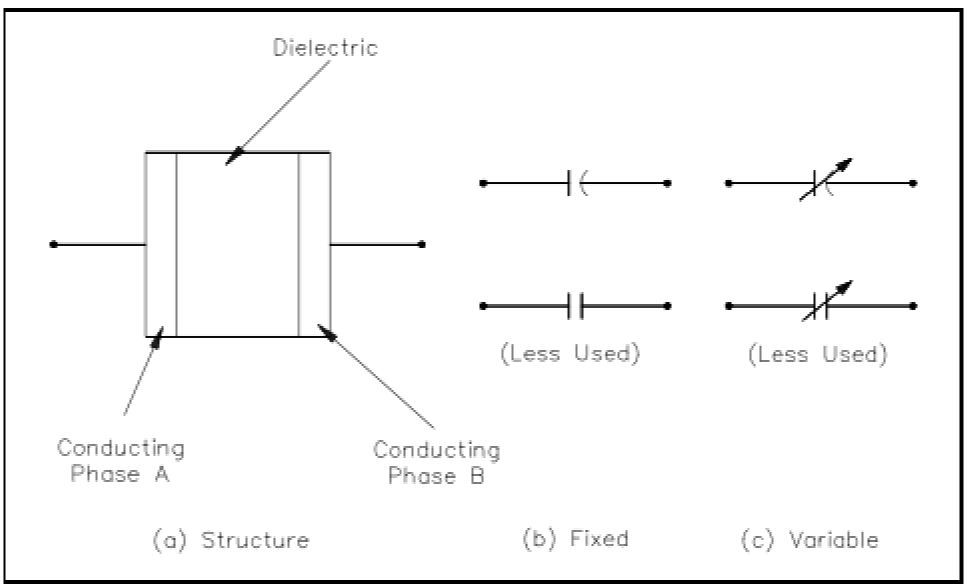

Figure 10 Capacitor and Symbols

The two conductor plates of the capacitor, shown in Figure 11 a, are electrically neutral, because there are as many positive as negative charges on each plate. The capacitor, therefore, has no charge.

Now, we connect a battery across the plates (Figure 11b). When the switch is closed (Figure 11c), the negative charges on Plate A are attracted to the positive side of the battery, while the positive charges on Plate B are attracted to the negative side of the battery. This movement of charges will continue until the difference in charge between Plate A and Plate B is equal to the voltage of the battery. This is now a "charged capacitor." Capacitors store energy as an electric field between the two plates.

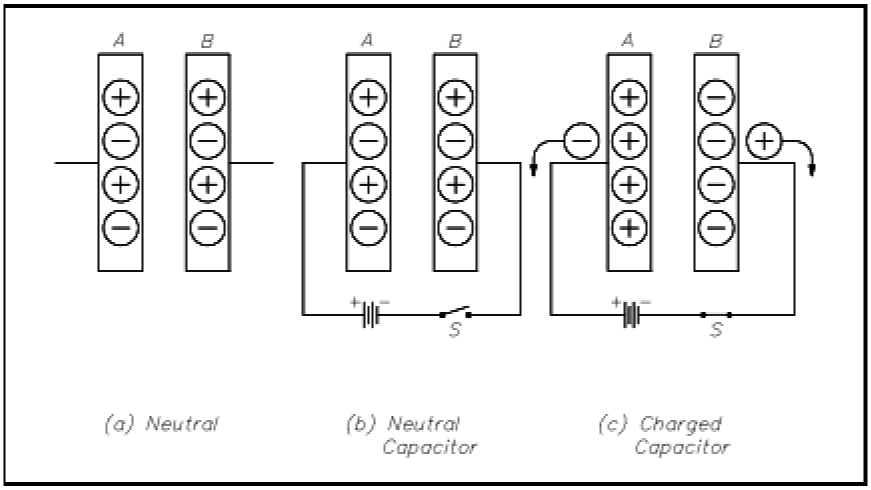

Figure 11 Charging a Capacitor

Because very few of the charges can cross between the plates, the capacitor will remain in the charged state even if the battery is removed. Because the charges on the opposing plates are attracted by one another, they will tend to oppose any changes in charge. In this manner, a capacitor will oppose any change in voltage felt across it.

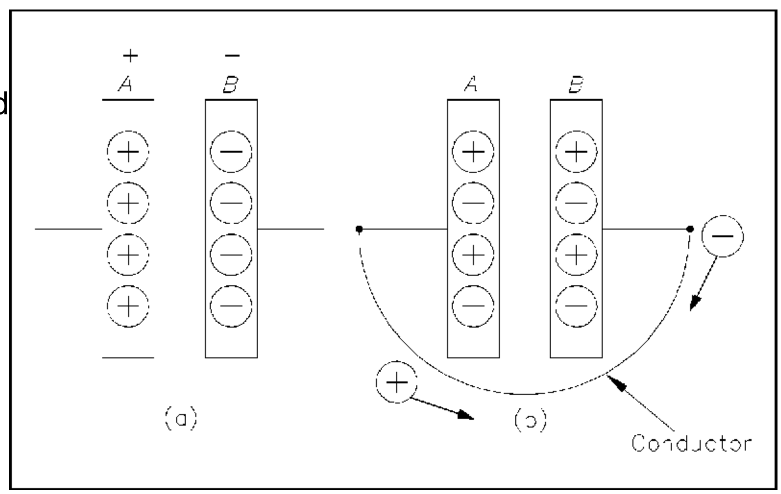

Figure 12 Discharging a Capacitor

If we place a conductor across the plates, electrons will find a path back to Plate A, and the charges will be neutralized again. This is now a "discharged" capacitor (Figure 12).

Capacitance

Capacitance is the ability to store an electrical charge. Capacitance is equal to the amount of charge that can be stored divided by the applied voltage, as shown in Equation (3-7).

$$C = Q / V \tag{3-7}$$

where

C	=	capacitance (F)
Q	=	amount of charge (C)
V	=	voltage (V)

The unit of capacitance is the farad (F). A farad is the capacitance that will store one coulomb of charge when one volt is applied across the plates of the capacitor.

The dielectric constant (K) describes the ability of the dielectric to store electrical energy. Air is used as a reference and is given a dielectric constant of 1. Therefore, the dielectric constant is unitless. Some other dielectric materials are paper, teflon, bakelite, mica, and ceramic.

The capacitance of a capacitor depends on three things.

1. Area of conductor plates
2. Separation between the plates
3. Dielectric constant of insulation material

Equation (3-8) illustrates the formula to find the capacitance of a capacitor with two parallel plates.

$$C = K \frac{A}{d} (8.85 \times 10^{-12}) \tag{3-8}$$

where

C	=	capacitance
K	=	dielectric constant
A	=	area
d	=	distance between the plates
8.85×10^{-12}	=	constant of proportionality

Example 1: Find the capacitance of a capacitor that stores 8C of charge at 4V.

$C = Q / V$

$C = 8 / 4$

$C = 2F$

Example 2: What is the charge taken on by a 5F capacitor at 2 volts?

$Q = C V$

$Q = (5F)(2V)$

$Q = 10C$

Example 3: What is the capacitance if the area of a two plate mica capacitor is 0.0050 m^2 and the separation between the plates is 0.04 m? The dielectric constant for mica is 7.

$$C = K \frac{A}{d} (8.85 \times 10^{-12})$$

$$C = 7 \frac{0.0050}{0.04} (8.85 \times 10^{-12})$$

$$= 7.74 \times 10^{-12} F$$

$$= 7.74 \rho F$$

Types of Capacitors

All commercial capacitors are named according to their dielectrics. The most common are air, mica, paper, and ceramic capacitors, plus the electrolytic type. These types of capacitors are compared in Table 1.

TABLE 1
Types of Capacitors

Dielectric	Construction	Capacitance Range
Air	Meshed plates	10 - 400ρF
Mica	Stacked Sheets	10 - 5000ρF
Paper	Rolled foil	0.001 - 1μF
Ceramic	Tubular	0.5 - 1600ρF
Disk	Tubular	0.002 - 0.1μF
Electrolytic	Aluminum	5 - 1000μF
Tantalum	Aluminum	0.01 - 300μF

Capacitors in Series and Parallel

Capacitors in series are combined like resistors in parallel. The total capacitance, c_T, of capacitors connected in series (Figure 13), is shown in Equation (3-9).

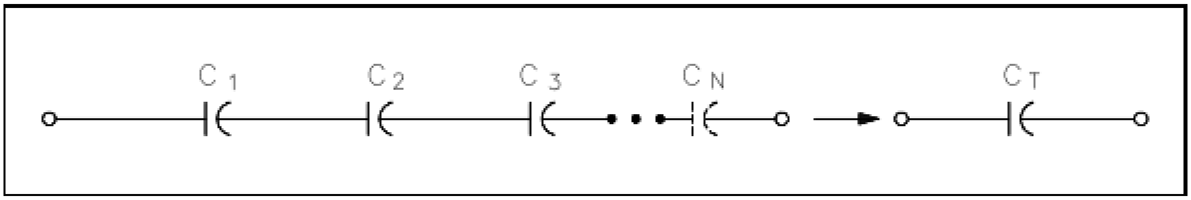

Figure 13 Capacitors Connected in Series

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \frac{1}{C_N} \tag{3-9}$$

When only two capacitors are in series, Equation (3-9) may be simplified as given in Equation (3-10). As shown in Equation (3-10), this is valid when there are only two capacitors in series.

$$C_T = \frac{C_1 C_2}{C_1 + C_2} \tag{3-10}$$

When all the capacitors in series are the same value, the total capacitance can be found by dividing the capacitor's value by the number of capacitors in series as given in Equation (3-11).

$$C_T = C / N \tag{3-11}$$

where

C = value of any capacitor in series

N = the number of capacitors in series with the same value.

Capacitors in parallel are combined like resistors in series. When capacitors are connected in parallel (Figure 14), the total capacitance, C_T, is the sum of the individual capacitances as given in Equation (3-12).

$$C_T = C_1 + C_2 + C_3 + \ldots + C_N \tag{3-12}$$

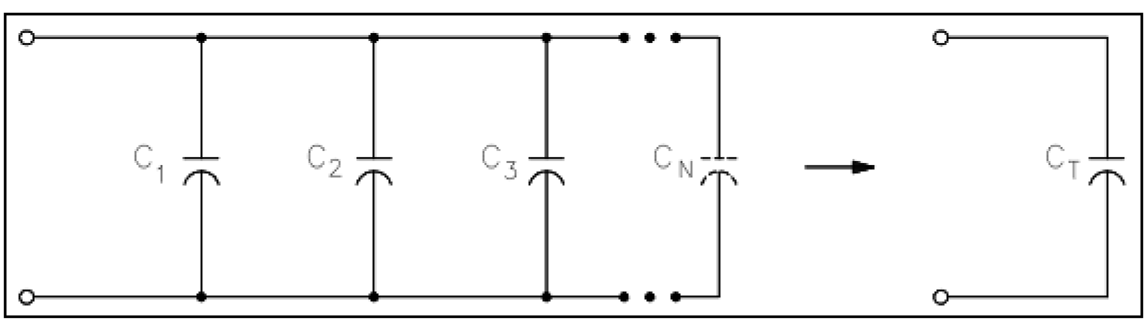

Figure 14 Capacitors Connected in Parallel

Example 1: Find the total capacitance of 3μF, 6μF, and 12μF capacitors connected in series (Figure 15).

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{3} + \frac{1}{6} + \frac{1}{12}$$

$$= \frac{4}{12} + \frac{2}{12} + \frac{1}{12}$$

$$= \frac{7}{12}$$

$$C_T = \frac{12}{7} = 1.7\mu\, f$$

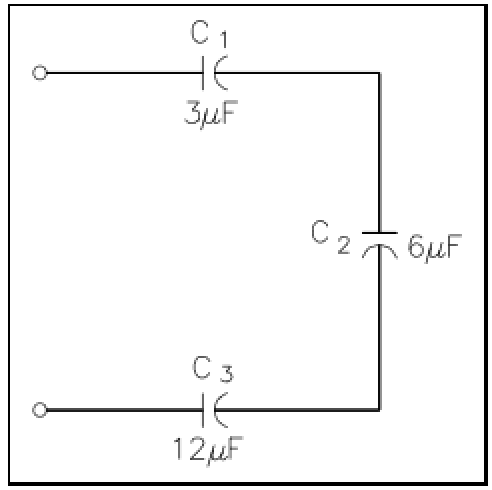

Figure 15 Example 1 - Capacitors Connected in Series

Example 2: Find the total capacitance and working voltage of two capacitors in series, when both have a value of 150 μF, 120 V (Figure 16).

$$C_T = \frac{C}{N}$$

$$= \frac{150}{2}$$

$$C_T = 75\mu\, f$$

Total voltage that can be applied across a group of capacitors in series is equal to the sum of the working voltages of the individual capacitors.

working voltage = 120 V + 120 V = 240 volts

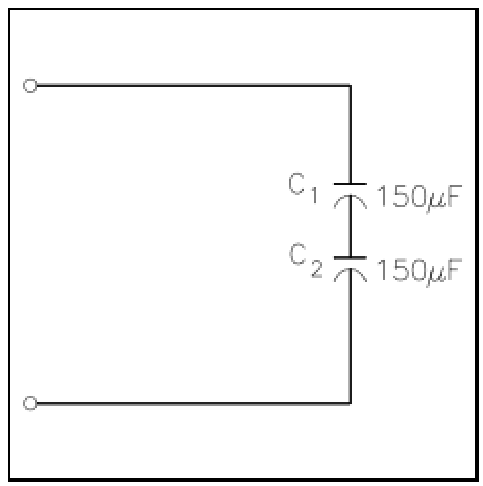

Figure 16 Example 2 - Capacitors Connected in Series

Example 3: Find the total capacitance of three capacitors in parallel, if the values are 15 μF-50 V, 10 μF-100 V, and 3 μF-150 V (Figure 17). What would be the working voltage?

$$C_T = C_1 + C_2 + C_3$$

$$= 15\mu F + 10\mu F + 3\mu F$$

$$C_T = 28\mu F$$

The working voltage of a group of capacitors in parallel is only as high as the lowest working voltage of an individual capacitor. Therefore, the working voltage of this combination is only 50 volts.

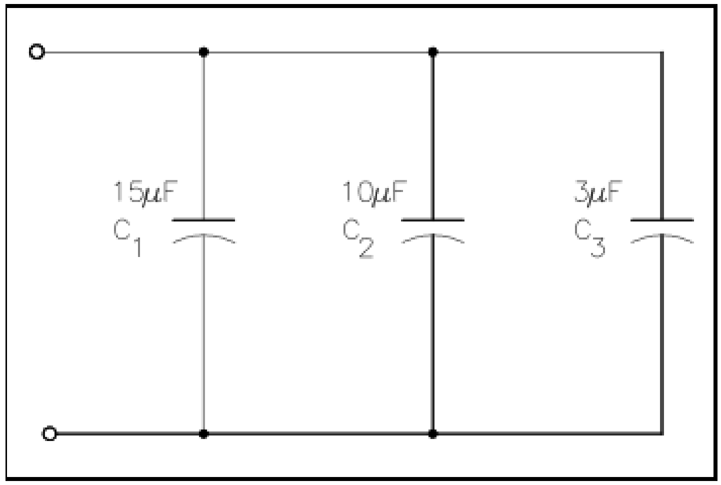

Figure 17 Example 3 - Capacitors Connected in Parallel

Capacitive Time Constant

When a capacitor is connected to a DC voltage source, it charges very rapidly. If no resistance was present in the charging circuit, the capacitor would become charged almost instantaneously. Resistance in a circuit will cause a delay in the time for charging a capacitor. The exact time required to charge a capacitor depends on the resistance (R) and the capacitance (C) in the charging circuit. Equation (3-13) illustrates this relationship.

$$T_c = R\,C \qquad (3\text{-}13)$$

where:

T_c = capacitive time constant (sec)

R = resistance (ohms)

C = capacitance (farad)

The capacitive time constant is the time required for the capacitor to charge to 63.2 percent of its fully charged voltage. In the following time constants, the capacitor will charge an additional 63.2 percent of the remaining voltage. The capacitor is considered fully charged after a period of five time constants (Figure 18).

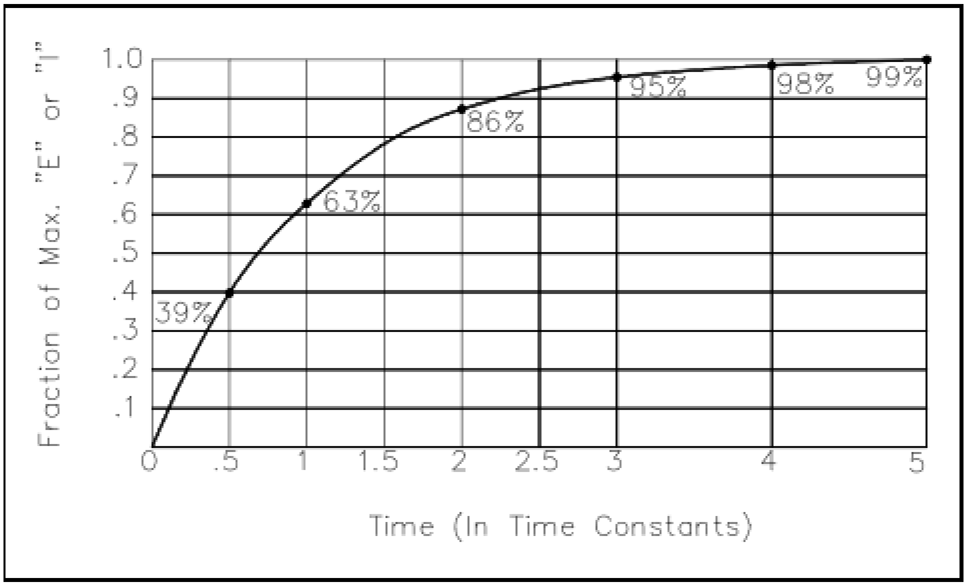

Figure 18 Capacitive Time Constant for Charging Capacitor

The capacitive time constant also shows that it requires five time constants for the voltage across a discharging capacitor to drop to its minimum value (Figure 19).

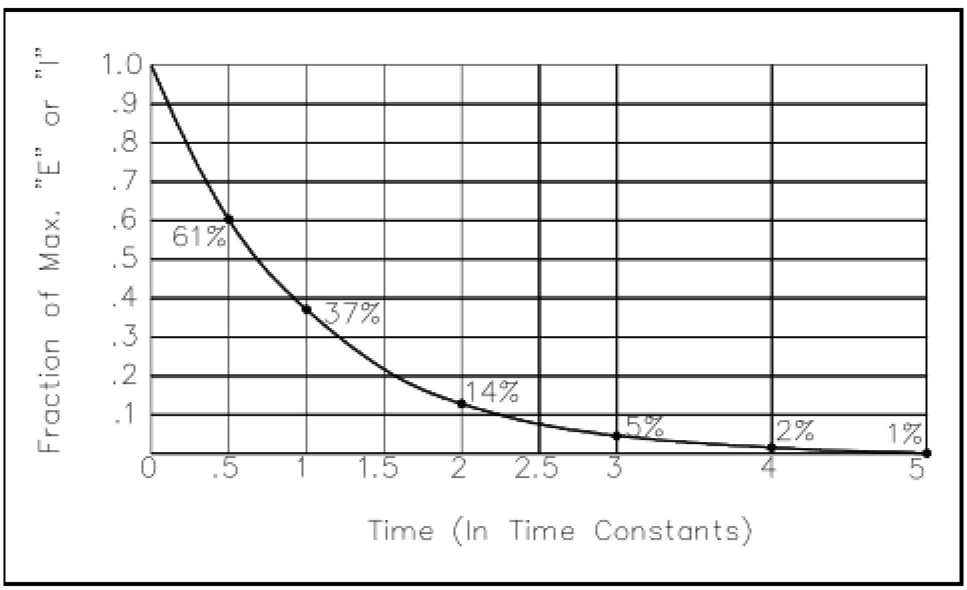

Figure 19 Capacitive Time Constant for Discharging Capacitor

Example: Find the time constant of a 100μF capacitor in series with a 100Ω resistor (Figure 20).

$$T_C = RC$$

$$T_C = (100\Omega)(100\mu F)$$

$$T_C = 0.01 \text{ seconds}$$

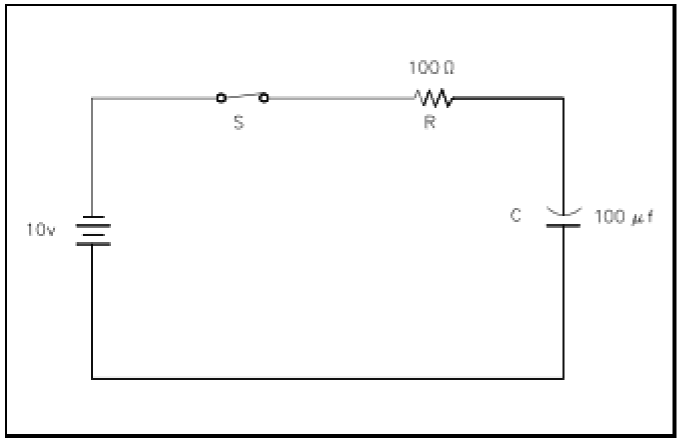

Figure 20 Example - Capacitive Time Constant

Summary

The important information on capacitors is summarized below.

Capacitance Summary

- A capacitor is constructed of two conductors (plates) separated by a dielectric.
- A capacitor will store energy in the form of an electric field caused by the attraction of the positively-charged particles in one plate to the negatively-charged particles in the other plate.
- The attraction of charges in the opposite plates of a capacitor opposes a change in voltage across the capacitor.
- Capacitors in series are combined like resistors in parallel.
- Capacitors in parallel are combined like resistors in series.
- The capacitive time constant is the time required for the capacitor to charge (or discharge) to 63.2 percent of its fully charged voltage.