DOE Fundamentals

# ELECTRICAL SCIENCE

## Module 8

## Basic AC Reactive Components

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# OBJECTIVES

## TERMINAL OBJECTIVE

1.0 Using the rules associated with inductors and capacitors, **DESCRIBE** the characteristics of these elements when they are placed in an AC circuit.

## ENABLING OBJECTIVES

1.1 **DESCRIBE** inductive reactance ($X_L$).

1.2 Given the operation frequency (f) and the value of inductance (L), **CALCULATE** the inductive reactance ($X_L$) of a simple circuit.

1.3 **DESCRIBE** the effect of the phase relationship between current and voltage in an inductive circuit.

1.4 **DRAW** a simple phasor diagram representing AC current (I) and voltage (E) in an inductive circuit.

1.5 **DEFINE** capacitive reactance ($X_c$).

1.6 Given the operating frequency (f) and the value of capacitance (C), **CALCULATE** the capacitive reactance ($X_c$) of a simple AC circuit.

1.7 **DESCRIBE** the effect on phase relationship between current (I) and voltage (E) in a capacitive circuit.

1.8 **DRAW** a simple phasor diagram representing AC current (I) and voltage (E) in a capacitive circuit.

1.9 **DEFINE** impedance (Z).

1.10 Given the values for resistance (R) and inductance (L) and a simple R-L series AC circuit, **CALCULATE** the impedance (Z) for that circuit.

1.11 Given the values for resistance (R) and capacitance (C) and a simple R-C series AC circuit, **CALCULATE** the impedance (Z) for that circuit.

1.12 Given a simple R-C-L series AC circuit and the values for resistance (R), inductive reactance ($X_L$), and capacitive reactance ($X_c$), **CALCULATE** the impedance (Z) for that circuit.

1.13 **STATE** the formula for calculating total current ($I_T$) in a simple parallel R-C-L AC circuit.

1.14 Given a simple R-C-L parallel AC circuit and the values for voltage ($V_T$), resistance (R), inductive reactance ($X_L$), and capacitive reactance ($X_c$), **CALCULATE** the impedance (Z) for that circuit.

1.15 **DEFINE** resonance.

1.16 Given the values of capacitance (C) and inductance (L), **CALCULATE** the resonant frequency.

1.17 Given a series R-C-L circuit at resonance, **DESCRIBE** the net reactance of the circuit.

1.18 Given a parallel R-C-L circuit at resonance, **DESCRIBE** the circuit output relative to current (I).

# INDUCTANCE

*Any device relying on magnetism or magnetic fields to operate is a form of inductor. Motors, generators, transformers, and coils are inductors. The use of an inductor in a circuit can cause current and voltage to become out-of-phase and inefficient unless corrected.*

EO 1.1 **DESCRIBE** inductive reactance ($X_L$).

EO 1.2 Given the operation frequency (f) and the value of inductance (L), **CALCULATE** the inductive reactance ($X_L$) of a simple circuit.

EO 1.3 **DESCRIBE** the effect of the phase relationship between current and voltage in an inductive circuit.

EO 1.4 **DRAW** a simple phasor diagram representing AC current (I) and voltage (E) in an inductive circuit.

## Inductive Reactance

In an inductive AC circuit, the current is continually changing and is continuously inducing an EMF. Because this EMF opposes the continuous change in the flowing current, its effect is measured in ohms. This opposition of the inductance to the flow of an alternating current is called *inductive reactance* ($X_L$). Equation (8-1) is the mathematical representation of the current flowing in a circuit that contains only inductive reactance.

$$I = \frac{E}{X_L} \tag{8-1}$$

where

I = effective current (A)

$X_L$ = inductive reactance ($\Omega$)

E = effective voltage across the reactance (V)

The value of $X_L$ in any circuit is dependent on the inductance of the circuit and on the rate at which the current is changing through the circuit. This rate of change depends on the frequency of the applied voltage. Equation (8-2) is the mathematical representation for $X_L$.

$$X_L = 2 \pi f L \tag{8-2}$$

where

$\pi$ = ~3.14

f = frequency (Hertz)

L = inductance (Henries)

The magnitude of an induced EMF in a circuit depends on how fast the flux that links the circuit is changing. In the case of self-induced EMF (such as in a coil), a counter EMF is induced in the coil due to a change in current and flux in the coil. This CEMF opposes any change in current, and its value at any time will depend on the rate at which the current and flux are changing at that time. In a purely inductive circuit, the resistance is negligible in comparison to the inductive reactance. The voltage applied to the circuit must always be equal and opposite to the EMF of self-induction.

## Voltage and Current Phase Relationships in an Inductive Circuit

As previously stated, any change in current in a coil (either a rise or a fall) causes a corresponding change of the magnetic flux around the coil. Because the current changes at its maximum rate when it is going through its zero value at 90° (point b on Figure 1) and 270° (point d), the flux change is also the greatest at those times. Consequently, the self-induced EMF in the coil is at its maximum (or minimum) value at these points, as shown in Figure 1. Because the current is not changing at the point when it is going through its peak value at 0° (point a), 180° (point c), and 360° (point e), the flux change is zero at those times. Therefore, the self-induced EMF in the coil is at its zero value at these points.

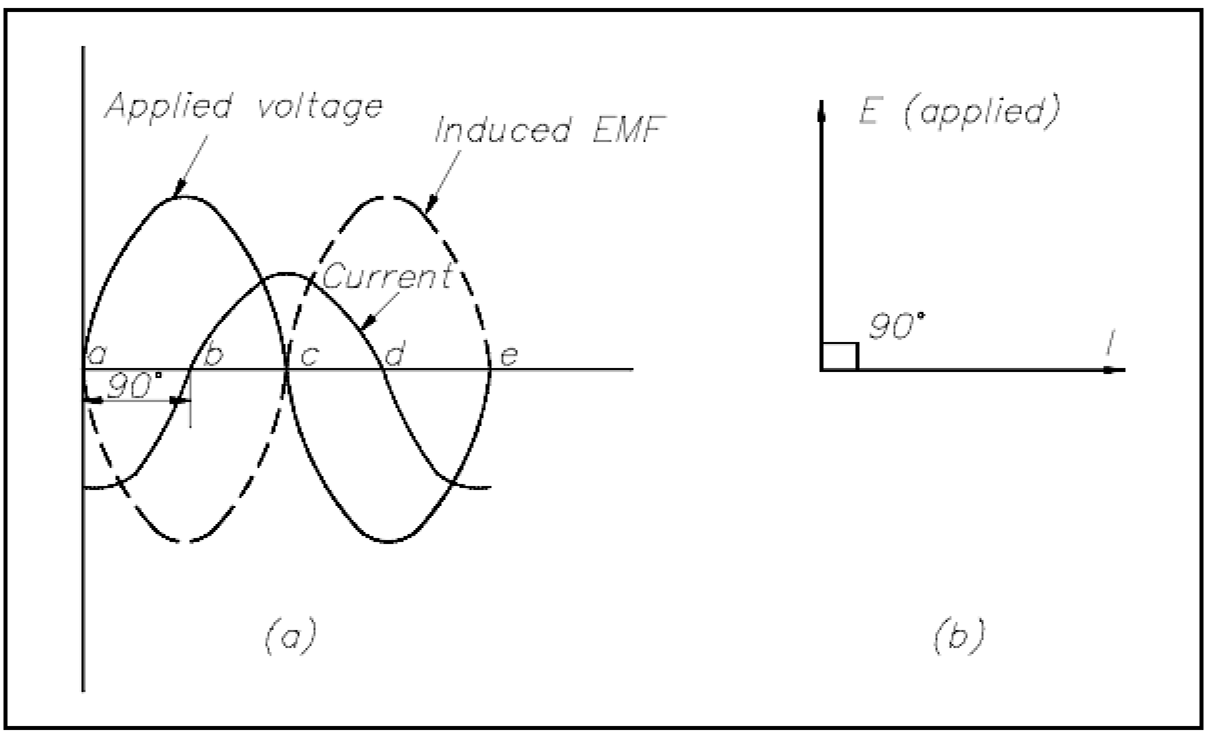


Figure 1 Current, Self-Induced EMF, and Applied Voltage in an Inductive Circuit

According to Lenz's Law (refer to Module 1, Basic Electrical Theory), the induced voltage always opposes the change in current. Referring to Figure 1, with the current at its maximum negative value (point a), the induced EMF is at a zero value and falling. Thus, when the current rises in a positive direction (point a to point c), the induced EMF is of opposite polarity to the applied voltage and opposes the rise in current. Notice that as the current passes through its zero value (point b) the induced voltage reaches its maximum negative value. With the current now at its maximum positive value (point c), the induced EMF is at a zero value and rising. As the current is falling toward its zero value at 180° (point c to point d), the induced EMF is of the same polarity as the current and tends to keep the current from falling. When the current reaches a zero value, the induced EMF is at its maximum positive value. Later, when the current is increasing from zero to its maximum negative value at 360° (point d to point e), the induced voltage is of the opposite polarity as the current and tends to keep the current from increasing in the negative direction. Thus, the induced EMF can be seen to lag the current by 90°.

The value of the self-induced EMF varies as a sine wave and lags the current by 90°, as shown in Figure 1. The applied voltage must be equal and opposite to the self-induced EMF at all times; therefore, the current lags the applied voltage by 90° in a purely inductive circuit.

If the applied voltage (E) is represented by a vector rotating in a counterclockwise direction (Figure 1b), then the current can be expressed as a vector that is lagging the applied voltage by 90°. Diagrams of this type are referred to as *phasor diagrams.*

Example: A 0.4 H coil with negligible resistance is connected to a 115V, 60 Hz power source (see Figure 2). Find the inductive reactance of the coil and the current through the circuit. Draw a phasor diagram showing the phase relationship between current and applied voltage.

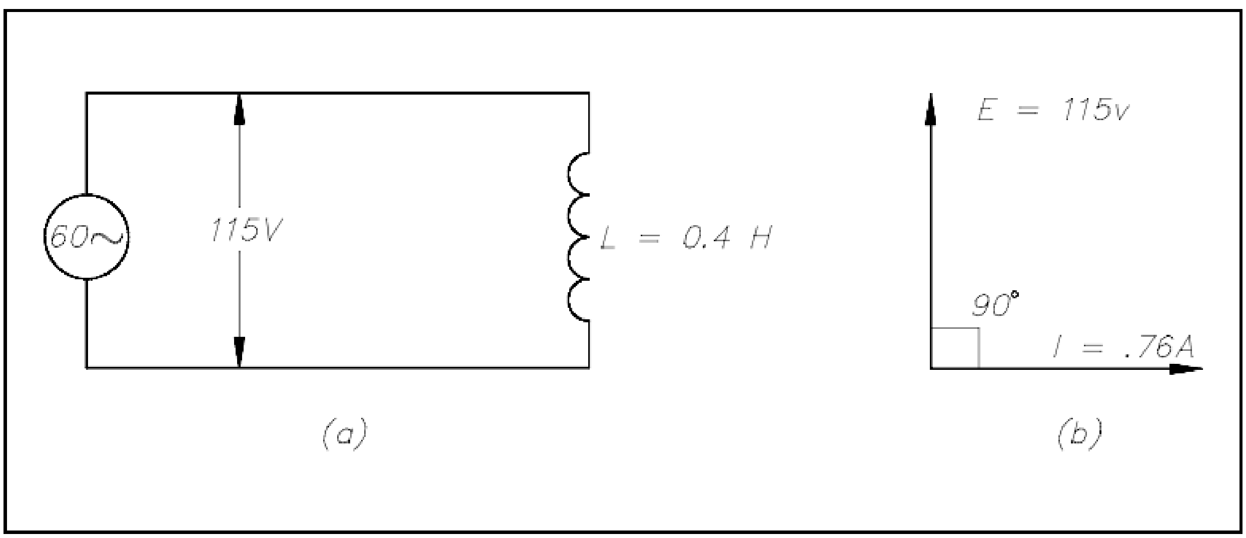


Figure 2 Coil Circuit and Phasor Diagram

Solution:

1. Inductive reactance of the coil

$$X_L = 2 \pi f L$$
$$= (2)(3.14)(60)(0.4)$$
$$X_L = 150.7\Omega$$

2. Current through the circuit

$$I = \frac{E}{X_L}$$
$$I = \frac{115}{150.7}$$
$$I = 0.76 \text{ amps}$$

3. Draw a phasor diagram showing the phase relationship between current and applied voltage.

   Phasor diagram showing the current lagging voltage by 90° is drawn in Figure 2b.

## Summary

Inductive reactance is summarized below.

**Inductive Reactance Summary**

- Opposition to the flow of alternating current caused by inductance is called Inductive Reactance ($X_L$).
- The formula for calculating $X_L$ is:

  $X_L = 2 \pi f L$
- Current (I) lags applied voltage (E) in a purely inductive circuit by 90° phase angle.
- The phasor diagram shows the applied voltage (E) vector leading (above) the current (I) vector by the amount of the phase angle differential due to the relationship between voltage and current in an inductive circuit.

# CAPACITANCE

*There are many natural causes of capacitance in AC power circuits, such as transmission lines, fluorescent lighting, and computer monitors. Normally, these are counteracted by the inductors previously discussed. However, where capacitors greatly outnumber inductive devices, we must calculate the amount of capacitance to add or subtract from an AC circuit by artificial means.*

EO 1.5 **DEFINE** capacitive reactance ($X_c$).

EO 1.6 Given the operating frequency (f) and the value of capacitance (C), **CALCULATE** the capacitive reactance ($X_c$) of a simple AC circuit.

EO 1.7 **DESCRIBE** the effect on phase relationship between current (I) and voltage (E) in a capacitive circuit.

EO 1.8 **DRAW** a simple phasor diagram representing AC current (I) and voltage (E) in a capacitive circuit.

## Capacitors

The variation of an alternating voltage applied to a capacitor, the charge on the capacitor, and the current flowing through the capacitor are represented by Figure 3.

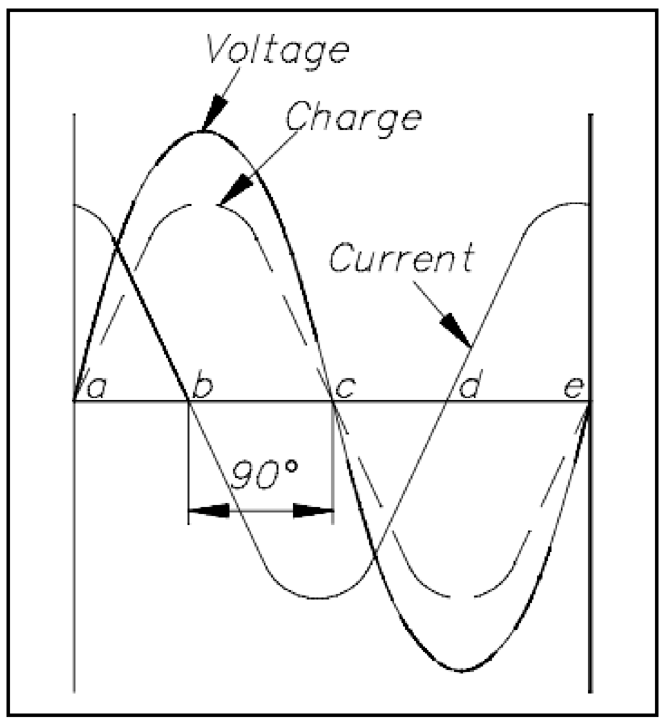

Figure 3 Voltage, Charge, and Current in a Capacitor

The current flow in a circuit containing capacitance depends on the rate at which the voltage changes. The current flow in Figure 3 is greatest at points a, c, and e. At these points, the voltage is changing at its maximum rate (i.e., passing through zero). Between points a and b, the voltage and charge are increasing, and the current flow is into the capacitor, but decreasing in value. At point b, the capacitor is fully charged, and the current is zero. From points b to c, the voltage and charge are decreasing as the capacitor discharges, and its current flows in a direction opposite to the voltage. From points c to d, the capacitor begins to charge in the opposite direction, and the voltage and current are again in the same direction.

At point d, the capacitor is fully charged, and the current flow is again zero. From points d to e, the capacitor discharges, and the flow of current is opposite to the voltage. Figure 3 shows the current leading the applied voltage by 90°. In any purely capacitive circuit, current leads applied voltage by 90°.

## Capacitive Reactance

*Capacitive reactance* is the opposition by a capacitor or a capacitive circuit to the flow of current. The current flowing in a capacitive circuit is directly proportional to the capacitance and to the rate at which the applied voltage is changing. The rate at which the applied voltage is changing is determined by the frequency of the supply; therefore, if the frequency of the capacitance of a given circuit is increased, the current flow will increase. It can also be said that if the frequency or capacitance is increased, the opposition to current flow decreases; therefore, capacitive reactance, which is the opposition to current flow, is inversely proportional to frequency and capacitance. Capacitive reactance $X_c$, is measured in ohms, as is inductive reactance. Equation (8-3) is a mathematical representation for capacitive reactance.

$$X_C = \frac{1}{2 \pi f C} \tag{8-3}$$

where

| | | |
|---|---|---|
| f | = | frequency (Hz) |
| π | = | ~3.14 |
| C | = | capacitance (farads) |

Equation (8-4) is the mathematical representation of capacitive reactance when capacitance is expressed in microfarads (μF).

$$X_C = \frac{1,000,000}{2 \pi f C} \tag{8-4}$$

Equation (8-5) is the mathematical representation for the current that flows in a circuit with only capacitive reactance.

$$I = \frac{E}{X_C} \tag{8-5}$$

where

| | | |
|---|---|---|
| I | = | effective current (A) |
| E | = | effective voltage across the capacitive reactance (V) |
| $X_c$ | = | capacitive reactance (Ω) |

Example: A 10μF capacitor is connected to a 120V, 60Hz power source (see Figure 4). Find the capacitive reactance and the current flowing in the circuit. Draw the phasor diagram.

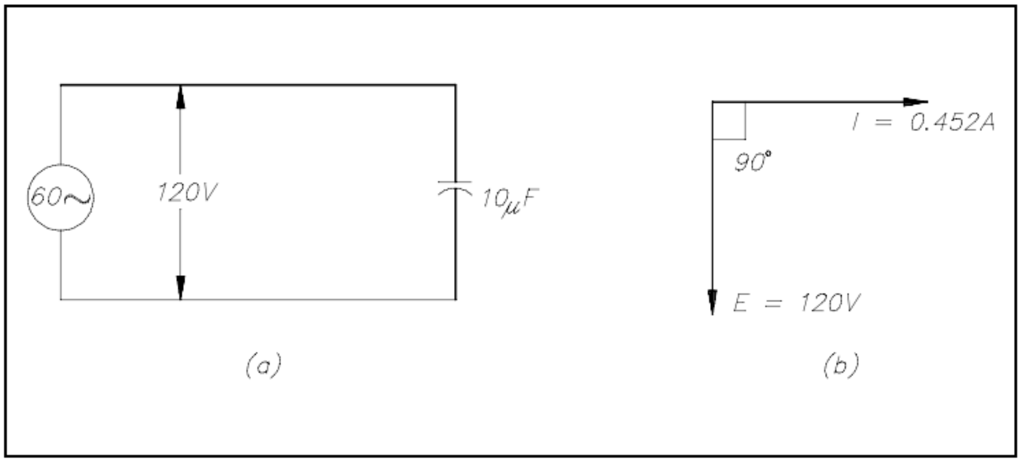


Figure 4 Circuit and Phasor Diagram

Solution:

1. Capacitive reactance

$$X_C = \frac{1,000,000}{2 \pi f C}$$

$$= \frac{1,000,000}{(2)(3.14)(60)(10)}$$

$$= \frac{1,000,000}{3768}$$

$$X_C = 265.4\Omega$$

2. Current flowing in the circuit

| | |
|---|---|
| $I =$ | $\frac{E}{X_C}$ |
| $=$ | $\frac{120}{265.4}$ |
| $I =$ | 0.452 amps |

3. Phasor diagram showing current leading voltage by 90° is drawn in Figure 4b.

## Summary

Capacitive reactance is summarized below.

**Capacitive Reactance Summary**

- Opposition to the flow of alternating current caused by capacitance is called capacitive reactance ($X_c$).
- The formula for calculating $X_c$ is:

  $$X_C = 1 / 2 \pi f C$$

- Current (I) leads applied voltage by 90° in a purely capacitive circuit.
- The phasor diagram shows the applied voltage (E) vector leading (below) the current (I) vector by the amount of the phase angle differential due to the relationship between voltage and current in a capacitive circuit.

# IMPEDANCE

*Whenever inductive and capacitive components are used in an AC circuit, the calculation of their effects on the flow of current is important.*

EO 1.9 **DEFINE** impedance (Z).

EO 1.10 Given the values for resistance (R) and inductance (L) and a simple R-L series AC circuit, **CALCULATE** the impedance (Z) for that circuit.

EO 1.11 Given the values for resistance (R) and capacitance (C) and a simple R-C series AC circuit, **CALCULATE** the impedance (Z) for that circuit.

EO 1.12 Given a simple R-C-L series AC circuit and the values for resistance (R), inductive reactance ($X_L$), and capacitive reactance ($X_c$), **CALCULATE** the impedance (Z) for that circuit.

EO 1.13 **STATE** the formula for calculating total current ($I_T$) in a simple parallel R-C-L AC circuit.

EO 1.14 Given a simple R-C-L parallel AC circuit and the values for voltage ($V_T$), resistance (R), inductive reactance ($X_L$), and capacitive reactance ($X_c$), **CALCULATE** the impedance (Z) for that circuit.

## Impedance

No circuit is without some resistance, whether desired or not. Resistive and reactive components in an AC circuit oppose current flow. The total opposition to current flow in a circuit depends on its resistance, its reactance, and the phase relationships between them. *Impedance* is defined as the total opposition to current flow in a circuit. Equation (8-6) is the mathematical representation for the magnitude of impedance in an AC circuit.

$$Z = \sqrt{R^2 + X^2} \qquad (8\text{-}6)$$

where

Z = impedance (Ω)

R = resistance (Ω)

X = net reactance (Ω)

The relationship between resistance, reactance, and impedance is shown in Figure 5.

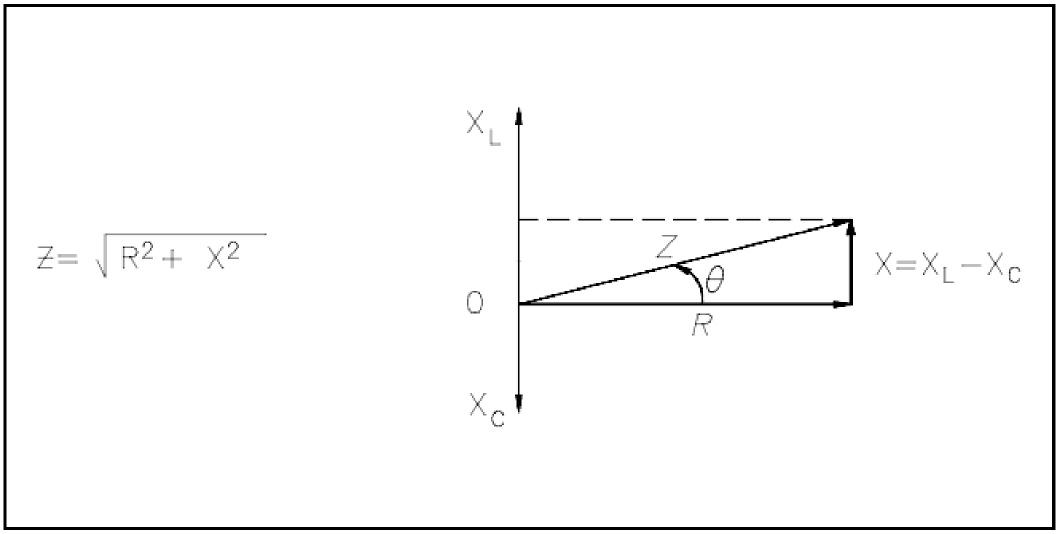


Figure 5 Relationship Between Resistance, Reactance, and Impedance

The current through a certain resistance is always in phase with the applied voltage. Resistance is shown on the zero axis. The current through an inductor lags applied voltage by 90'; inductive reactance is shown along the 90° axis. Current through a capacitor leads applied voltage by 90°; capacitive reactance is shown along the -90° axis. Net reactance in an AC circuit is the difference between inductive and capacitive reactance. Equation (8-7) is the mathematical representation for the calculation of net reactance when $X_L$ is greater than $X_c$.

$$X = X_L - X_c \tag{8-7}$$

where

$X$ = net reactance (Ω)

$X_L$ = inductive reactance (Ω)

$X_c$ = capacitive reactance (Ω)

Equation (8-8) is the mathematical representation for the calculation of net reactance when $X_c$ is greater than $X_L$.

$$X = X_c - X_L \tag{8-8}$$

Impedance is the vector sum of the resistance and net reactance (X) in a circuit, as shown in Figure 5. The angle Θis the phase angle and gives the phase relationship between the applied voltage and the current. Impedance in an AC circuit corresponds to the resistance of a DC circuit. The voltage drop across an AC circuit element equals the current times the impedance. Equation (8-9) is the mathematical representation of the voltage drop across an AC circuit.

$$V = I\,Z \tag{8-9}$$

where

| | | |
|---|---|---|
| V | = | voltage drop (V) |
| I | = | current (A) |
| Z | = | impedance (Ω) |

The phase angle Θ gives the phase relationship between current and the voltage.

## Impedance in R-L Circuits

Impedance is the resultant of phasor addition of R and $X_L$. The symbol for impedance is Z. Impedance is the total opposition to the flow of current and is expressed in ohms. Equation (8-10) is the mathematical representation of the impedance in an RL circuit.

$$Z = \sqrt{R^2 + X_L^2} \tag{8-10}$$

Example: If a 100Ω resistor and a 60Ω $X_L$ are in series with a 115V applied voltage (Figure 6), what is the circuit impedance?

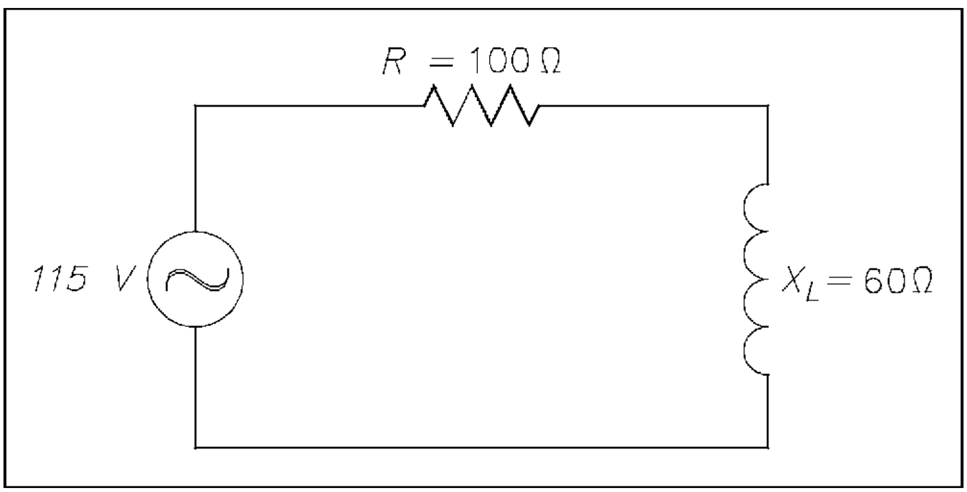


Figure 6 Simple R-L Circuit

Solution:

$$Z = \sqrt{R^2 + X_L^2}$$

$$Z = \sqrt{100^2 + 60^2}$$

$$Z = \sqrt{10{,}000 + 3600}$$

$$Z = \sqrt{13{,}600}$$

$$Z = 116.6\Omega$$

## Impedance in R-C Circuits

In a capacitive circuit, as in an inductive circuit, impedance is the resultant of phasor addition of R and $X_c$. Equation (8-11) is the mathematical representation for impedance in an R-C circuit.

$$Z = \sqrt{R^2 + X_C^2} \tag{8-11}$$

Example: A 50Ω $X_C$ and a 60Ω resistance are in series across a 110 VAC source (Figure 7). Calculate the impedance.

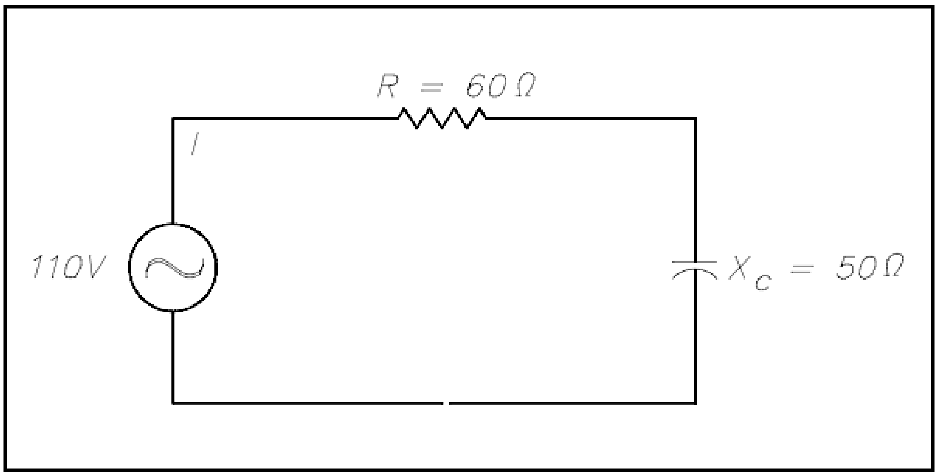


Figure 7 Simple R-C Circuit

Solution:

$$Z = \sqrt{R^2 + X_C^2}$$

$$Z = \sqrt{60^2 + 50^2}$$

$$Z = \sqrt{3600 + 2500}$$

$$Z = \sqrt{6100}$$

$$Z = 78.1\Omega$$

## Impedance in R-C-L Circuits

Impedance in an R-C-L series circuit is equal to the phasor sum of resistance, inductive reactance, and capacitive reactance (Figure 8).

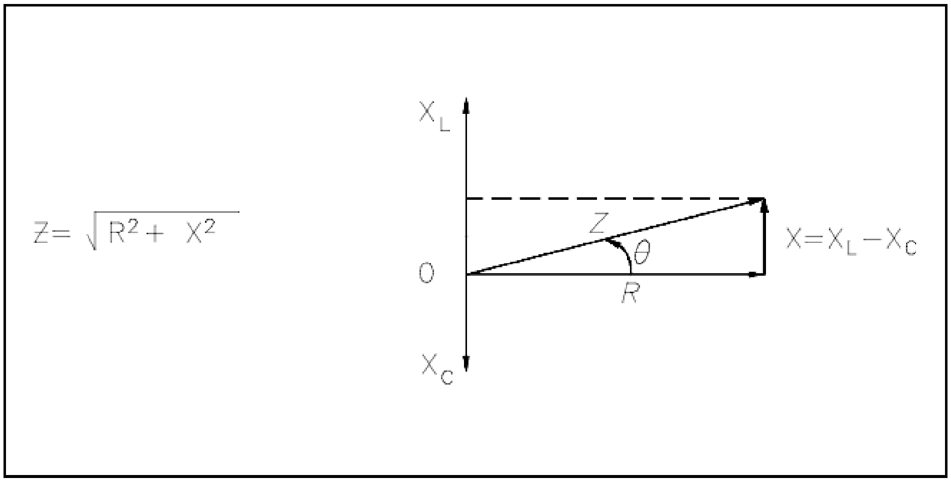


Figure 8 Series R-C-L Impedance-Phasor

Equations (8-12) and (8-13) are the mathematical representations of impedance in an R-C-L circuit. Because the difference between $X_L$ and $X_c$ is squared, the order in which the quantities are subtracted does not affect the answer.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad (8\text{-}12)$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2} \quad (8\text{-}13)$$

Example: Find the impedance of a series R-C-L circuit, when R = 6Ω, $X_L$ = 20Ω, and $X_c$ = 10Ω (Figure 9).

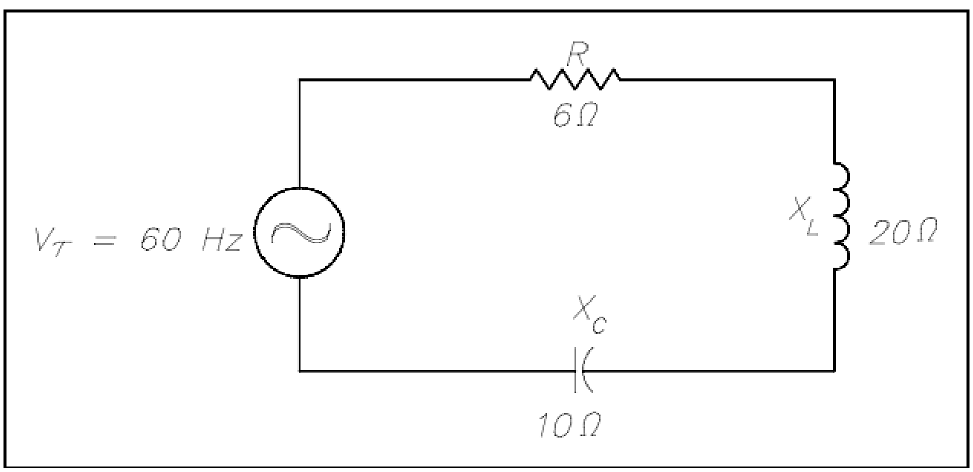


Figure 9 Simple R-C-L Circuit

Solution:

$$Z = \sqrt{R^2 + (X_L - X_C)^2\ })$$

$$Z = \sqrt{6^2 + (20 - 10)^2\ })$$

$$Z = \sqrt{6^2 + (10)^2\ })$$

$$Z = \sqrt{36 + 100\ })$$

$$Z = \sqrt{136}\,)$$
$$Z = 11.66\Omega)$$

Impedance in a parallel R-C-L circuit equals the voltage divided by the total current. Equation (8-14) is the mathematical representation of the impedance in a parallel R-C-L circuit.

$$Z_T = \frac{V_T}{I_T} \tag{8-14}$$

where

$Z_T$ = total impedance (Ω)

$V_T$ = total voltage (V)

$I_T$ = total current (A)

Total current in a parallel R-C-L circuit is equal to the square root of the sum of the squares of the current flows through the resistance, inductive reactance, and capacitive reactance branches of the circuit. Equations (8-15) and (8-16) are the mathematical representations of total current in a parallel R-C-L circuit. Because the difference between $I_L$ and $I_c$ is squared, the order in which the quantities are subtracted does not affect the answer.

$$I_T = \sqrt{I_R^2 + (I_C - I_L)^2}\,) \tag{8-15}$$

$$I_T = \sqrt{I_R^2 + (I_L - I_C)^2}\,) \tag{8-16}$$

where

$I_T$ = total current (A)

$I_R$ = current through resistance leg (A)

$I_C$ = current through capacitive reactance leg (A)

$I_L$ = current through inductive resistance leg (A)

Example: A 200Ω resistor, a 100Ω $X_L$, and an 80Ω $X_c$ are placed in parallel across a 120V AC source (Figure 10). Find: (1) the branch currents, (2) the total current, and (3) the impedance.

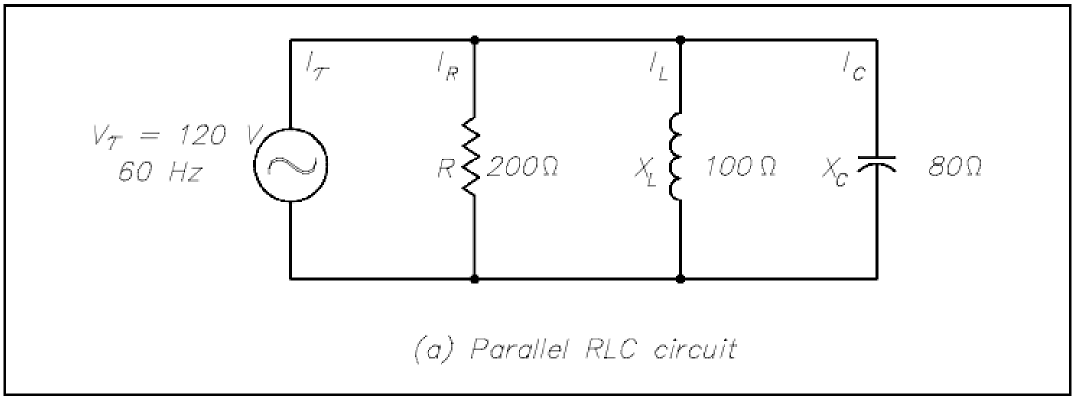


Figure 10 Simple Parallel R-C-L Circuit

Solution:

1. Branch currents:

$$I_R = \frac{V_T}{R} = \frac{120}{200} \qquad I_R = 0.6a$$

$$I_L = \frac{V_T}{X_L} = \frac{120}{100} \qquad I_L = 1.2a$$

$$I_C = \frac{V_T}{X_C} = \frac{120}{80} \qquad I_C = 1.5a$$

2. Total current:

$$I_T = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$I_T = \sqrt{0.6^2 + (1.5 - 1.2)^2}$$

$$I_T = \sqrt{0.6^2 + (0.3)^2}$$

$$I_T = \sqrt{0.36 + 0.09}$$

$$I_T = \sqrt{0.45} = 0.671A$$

3. Impedance

$$Z = \frac{V_T}{I_T}$$

$$Z = \frac{120}{0.671}$$

$$Z = 178.8\Omega$$

## Summary

Impedance is summarized below.

**Impedance Summary**

- Impedance (Z) is the total opposition to current flow in an AC circuit.
- The formula for impedance in a series AC circuit is:

  $$Z = \sqrt{R^2 + X^2}$$

- The formula for impedance in a parallel R-C-L circuit is:

  $$Z = \sqrt{R^2 + (X_L - X_C)^2}\ )$$

- The formulas for finding total current ($I_T$) in a parallel R-C-L circuit are:

  when $I_C > I_L$, $I_T = \sqrt{I_R^2 + (I_C - I_L)^2}$

  when $I_L > I_C$, $I_T = \sqrt{I_R^2 + (I_L - I_C)^2}$

# RESONANCE

*In the chapters on inductance and capacitance we have learned that both conditions are reactive and can provide opposition to current flow, but for opposite reasons. Therefore, it is important to find the point where inductance and capacitance cancel one another to achieve efficient operation of AC circuits.*

EO 1.15 **DEFINE** resonance.

EO 1.16 Given the values of capacitance (C) and inductance (L), **CALCULATE** the resonant frequency.

EO 1.17 Given a series R-C-L circuit at resonance, **DESCRIBE** the net reactance of the circuit.

EO 1.18 Given a parallel R-C-L circuit at resonance, **DESCRIBE** the circuit output relative to current (I).

## Resonant Frequency

*Resonance* occurs in an AC circuit when inductive reactance and capacitive reactance are equal to one another: $X_L = X_c$ When this occurs, the total reactance, $X = X_L - X_c$ becomes zero and the impendence is totally resistive. Because inductive reactance and capacitive reactance are both dependent on frequency, it is possible to bring a circuit to resonance by adjusting the frequency of the applied voltage. Resonant frequency ($f_{Res}$) is the frequency at which resonance occurs, or where $X_L = X_c$. Equation (8-14) is the mathematical representation for resonant frequency.

$$f_{Res} = \frac{1}{2\pi\sqrt{L\,C}} \tag{8-14}$$

where

$f_{Res}$ = resonant frequency (Hz)

L = inductance (H)

C = capacitance (f)

## Series Resonance

In a series R-C-L circuit, as in Figure 9, at resonance the net reactance of the circuit is zero, and the impedance is equal to the circuit resistance; therefore, the current output of a series resonant circuit is at a maximum value for that circuit and is determined by the value of the resistance. (Z=R)

$$I = \frac{V_T}{Z_T} = \frac{V_T}{R}$$

## Parallel Resonance

Resonance in a parallel R-C-L circuit will occur when the reactive current in the inductive branches is equal to the reactive current in the capacitive branches (or when $X_L = X_c$). Because inductive and capacitive reactance currents are equal and opposite in phase, they cancel one another at parallel resonance.

If a capacitor and an inductor, each with negligible resistance, are connected in parallel and the frequency is adjusted such that reactances are exactly equal, current will flow in the inductor and the capacitor, but the total current will be negligible. The parallel C-L circuit will present an almost infinite impedance. The capacitor will alternately charge and discharge through the inductor. Thus, in a parallel R-C-L, as in Figure 10, the net current flow through the circuit is at minimum because of the high impendence presented by $X_L$ and $X_c$ in parallel.

## Summary

Resonance is summarized below.

**Resonance Summary**

- Resonance is a state in which the inductive reactance equals the capacitive reactance ($X_L = X_c$) at a specified frequency ($f_{Res}$).
- Resonant frequency is:

$$f_{Res} = \frac{1}{2\pi\sqrt{L\,C}}$$

- R-C-L series circuit at resonance is when net reactance is zero and circuit current output is determined by the series resistance of the circuit.
- R-C-L parallel circuit at resonance is when net reactance is maximum and circuit current output is at minimum